Demonstrating Negative Refraction

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Introduction

When electromagnetic waves transfer from one medium to another the difference in the speed of the wave in the new medium causes the wave to bend. The way the waves bend depends on the medium's index of refraction. All naturally occurring materials have been shown to have a positive index of refraction. [1] However, it is possible to construct artificial materials, called metamaterials, which based on their structure could have extraordinary properties such as a negative index of refraction. In this paper I will show how my partner and I built a metamaterial with a negative index of refraction and how to experimentally demonstrate this property. A typical way to demonstrate refraction is to shine electromagnetic waves on a prism and measure the angle of refraction. In our experiment we will construct a material with a dielectric permittivity, ε, and the magnetic permeability, μ, less than zero and shine linearly polarized planar microwaves on the prism, and then we will measure the angle of the refracted beam.

Theoretical Background

The way electromagnetic waves propagate within a material depends on the materials ε and μ. In order for a material to have a negative index of refraction ε and μ must have simultaneously negative values. Materials with ε and μ less than zero are called left handed materials as opposed to the usual right handed materials that have a positive index of refraction. The reason for this can be explained by Maxwell's equations for planer waves:

$$\boldsymbol{k} \times \boldsymbol{E} = \omega\mu\boldsymbol{H}, \tag{1}$$

$$\boldsymbol{k} \times \boldsymbol{H} = -\omega\varepsilon\boldsymbol{E}, \tag{2}$$

Where **E** is the electric field, **H** is the magnetic field, and **k** is the wave vector. For positive ε and μ, **E**, **H**, and **k** form a right handed system of vectors; however, if ε and μ are negative then equations (1) and (2) become

$$\boldsymbol{k} \times \boldsymbol{E} = -\omega|\mu|\boldsymbol{H}, \tag{3}$$

$$\boldsymbol{k} \times \boldsymbol{H} = \omega|\varepsilon|\boldsymbol{E}, \tag{4}$$

and **E**,**H**, and **k**, form a left handed system of vectors. Thus, materials are called right handed or left handed materials respectively.[2] [3]

From equations (3) and (4) we see why we need ε and μ to be negative to have negative refraction through a medium. The wave vector travels in the opposite direction as the energy flux so we get backwards wave propagation in a left handed material. Thus, from Snell's law,

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{-|\boldsymbol{k}_2|}{|\boldsymbol{k}_1|} \equiv \frac{n_2}{n_1} < 0, \tag{5}$$

Where $n_1$is the refractive index of a normal right handed material and n_2 is the refractive index of a left handed material. As n_1> 0, then n_2 must be negative. Thus, the refractive index for a left handed material must be[2]

$$n \equiv -\sqrt{\varepsilon\mu} < 0. \tag{6}$$

Aqcuiring negative $\boldsymbol{\varepsilon}$ and $\boldsymbol{\mu}$

In our experiment we will be shining our prism with linearly polarized plane waves. Therefore, ε and μ only need to be negative in the direction of the electric and magnetic field respectively. Thus, a way to acquire ε < 0 is to stack metallic plates parallel to the electric field separated by some distance, d. This way the plates act as a parallel plate waveguide. Then from the propagation constant, β, in a parallel plate waveguide where

$$\beta = \sqrt{k_0^2 - \left(\frac{\pi}{d}\right)^2}, \tag{7}$$

and k_0 is the free-space wave number, we can find the effective relative permittivity of the waveguide by

$$\varepsilon_r = \left(\frac{\beta}{k_0}\right)^2 = 1 - \left(\frac{\pi}{\mathrm{d}k_0}\right)^2 = 1 - \left(\frac{c}{2\,\mathrm{d}f}\right)^2, \tag{8}$$

where c is the speed of light in a vacuum and f is the operating frequency. Thus, as long as $f < \frac{c}{2d}$ then $\varepsilon_r < 0.$[2]

For our metamaterial to have μ negative in the direction of the magnetic field, we need to construct a planar array of split ring resonators (SRR) that will resonate slightly below our operating frequency. SRRs consist of metallic rings with splits in the rings. An alternating magnetic field perpendicular to the plane of the rings induces current in the rings. This causes the SRRs to act like an inductor-capacitor oscillator with a resonant frequency. An array of SRRs close to their resonant frequencies exhibit $\mu < 0.$[4]

Constructing our Prism

In order to construct our left handed metamaterial, my partner and I first created a SSR pattern in Inkscape[see last page]. We then printed this pattern onto glossy paper and transferred it onto a copper-clad printed circuit board by taping the sheet face down on the circuit board and ironing it with a conventional household iron for a few minutes. Next, we took the board and submerged it in ferric chloride in order to etch our SSR pattern. After about an hour the unwanted copper was eaten away. We then washed our boards in water and repeated this process two more times to create a total of three boards. We then placed each circuit board between pieces of foam poster board with a thickness of 6mm. Very thin copper-clad printed circuit board was glued to the top and the bottom of this structure in order to act as a parallel plate waveguide which will give us the negative value of ε [see Fig.1(b)]. We constructed three of these layers and then combined them to form our final metamaterial [see Fig. 1(a)]. Lastly, our metamaterial was cut to form a prism with the dimensions shown in Fig. 1(c).

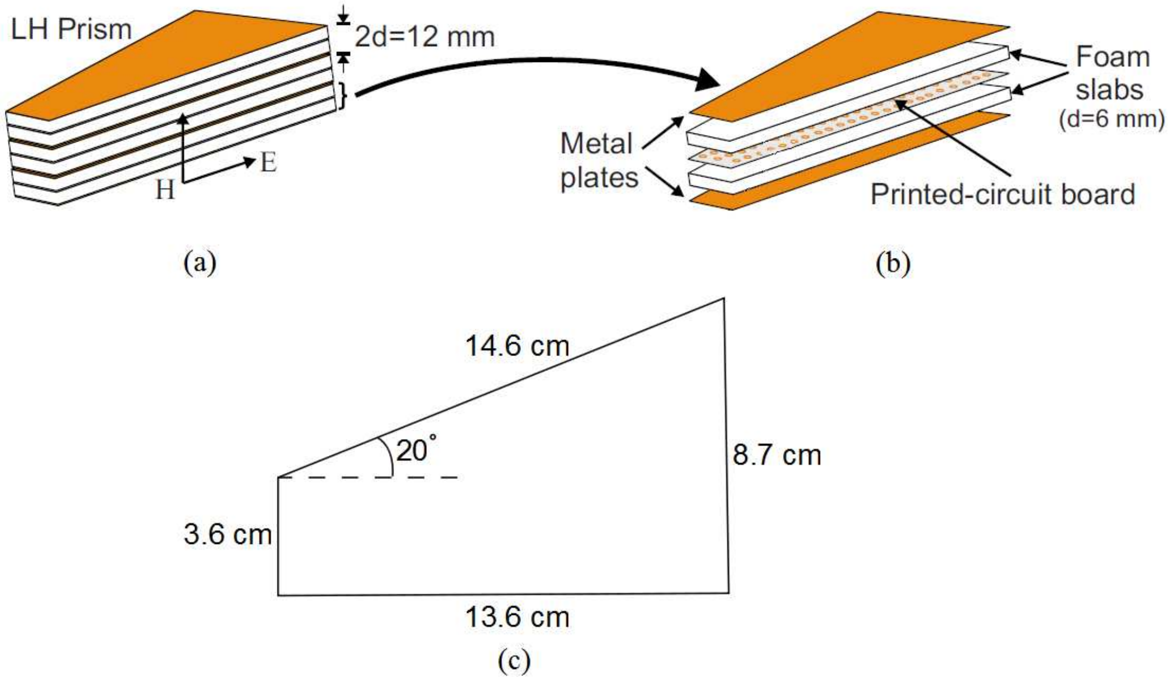


Fig. 1. (a) Our final left handed prism. (b) A detailed single layer of the prism. (c) The dimensions of our prism.

Experimental Setup

Fig. 2. Shows a picture of our experimental setup. A power supply is connected to a microwave emitter that emits microwaves with a wavelength that we calculated using a Michelson interferometer to be about 3.2 cm or a frequency of 9.5 GHz. Using wooden board we constructed a mounting device with a slot through it to mount our prism in. We covered the side of the mounting device facing the emitter with aluminum foil and also stood copper-clad boards to the left and the right of it to ensure no microwaves passed through. The mounting device was placed 30cm away from the microwave emitter. The microwave receiver was attached to a plastic rod that was fixed to the table yet free to rotate. The receiver was 61cm straight across from the emitter. A protractor was taped to the table so we could measure the angle the receiver was at. The receiver was also connected to a voltmeter. We then rotated the receiver from -40˚ to 40˚ at 5˚ increments and recorded our readings from the voltmeter. We did this with our left handed prism and also for a prism made of plastic.

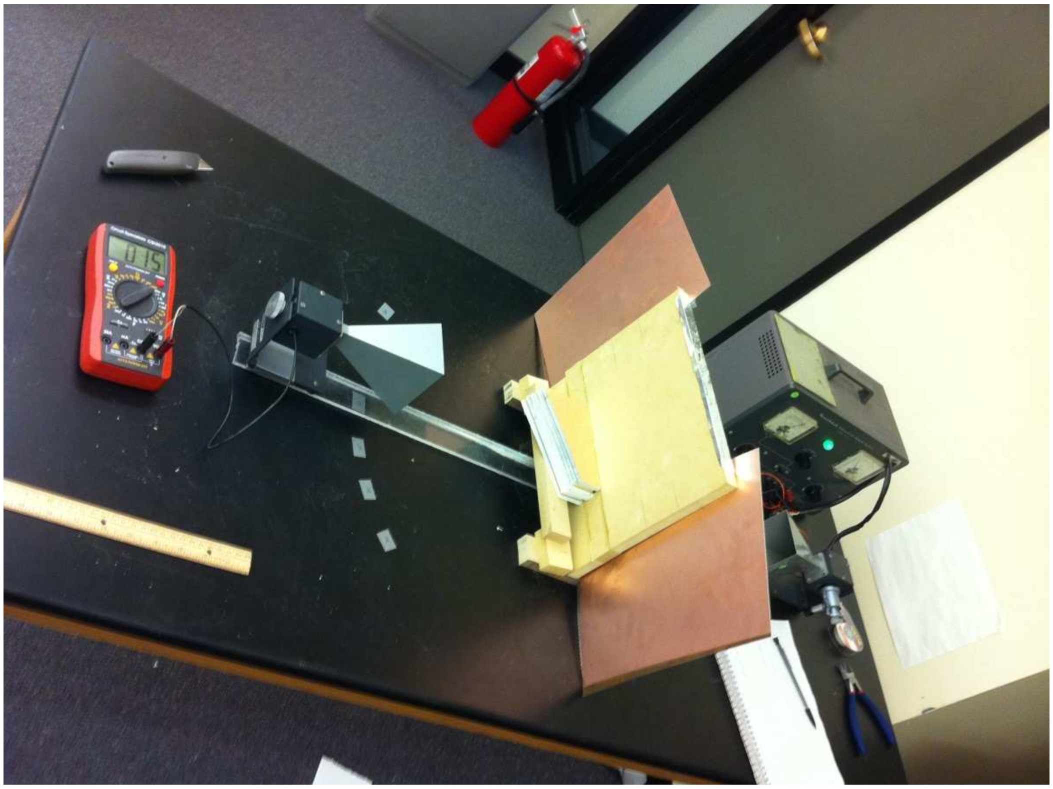
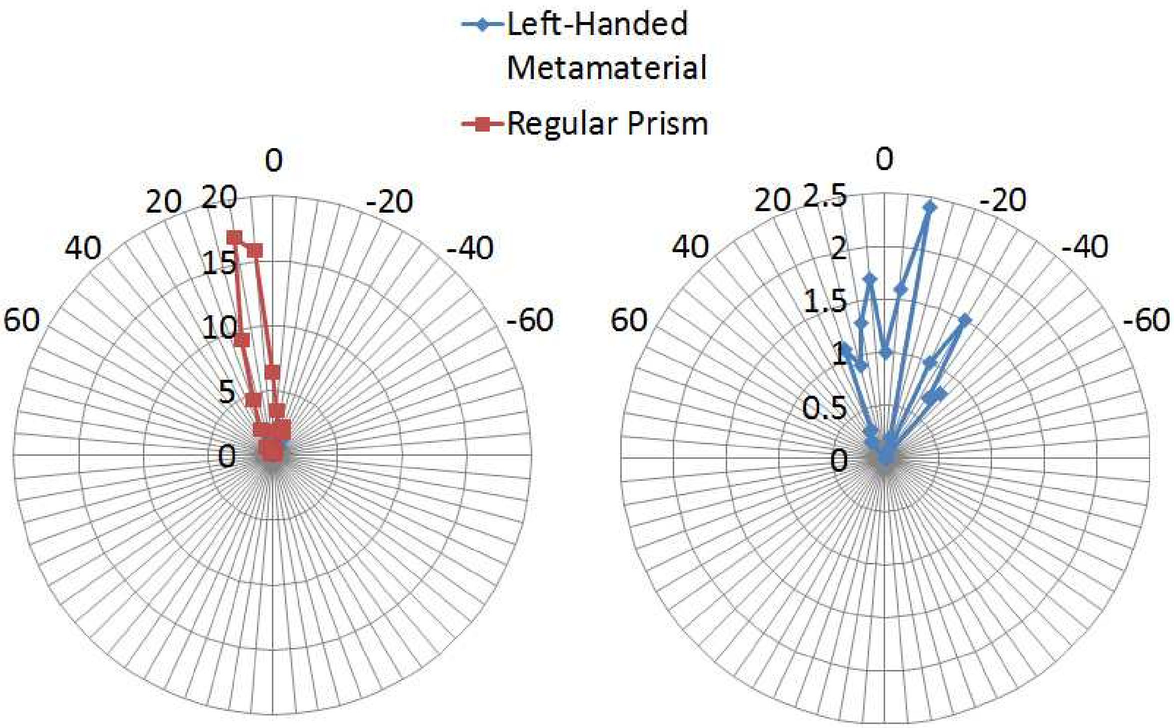


Fig. 2. Our experimental setup.

Results

Fig. 3. Shows the results from our experiment. All angles were measured with respect to the mounting device which is a positive 20˚ from the normal of the prism. It is clear that the plastic prism refracted normally like a right handed material. On the other hand, the left handed prism demonstrates strange refractive properties. The highest voltage reading for the left handed prism occurs at -10 ˚. This corresponds to refraction that is positive, but less than one. We also get a strong reading at degrees that correspond to normal refraction. Moreover, there is a strong reading at -30 ˚ which corresponds to negative refraction.



(a) (b)

Fig. 3. (a) The results from our prism made from plastic. (b) the results of our left handed prism.

Conclusion

Although we did not construct a prism that was completely negatively refractive, we did construct a prism that exhibited refractive phenomena that was atypical to the natural norm.

We could improve our results by finding a better way to make our printed circuit boards with our SRR pattern. We often times had rings on the board that would run together and places where the rings only partially transferred or did not transfer to the board at all. Also, we were unaware of how changing the size of our SRRs would effect their resonant frequency. We originally made our SRRs to resonate at a frequency of 10.5GHz. However, we could only produce waves of 9.5GHz so we had to guess and check to find the size of rings that worked the best. If we knew a way to find exactly what size we needed we could make the SRRs resonate better. Likewise, having a tunable microwave emitter would help us out as well.

References

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