

# Fractal Characteristics of Electrochemically Deposited Copper

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## Introduction

Fractals are found everywhere, in snowflakes, trees and the shore of continents. They are very useful to understand as they may define how a system will work or its possible structures and efficiency. Although, fractals have been widely researched in the past, the specifics to how the fractals form is not well understood. Such is the case with Copper fractals making them an interesting area of research.

## Fractals

Fractals are any self-similar object with a non-integral dimension. A self-similar object is any object that has large and small pattern repetitions throughout it. For example, in the picture below (figure 1) is a snowflake with large branches stemming from the center and smaller branches growing from those. Examining the smaller branches, one can see the resemblance to the large branches in that the small branches have even smaller branches growing off of them in the same pattern.

Figure 1

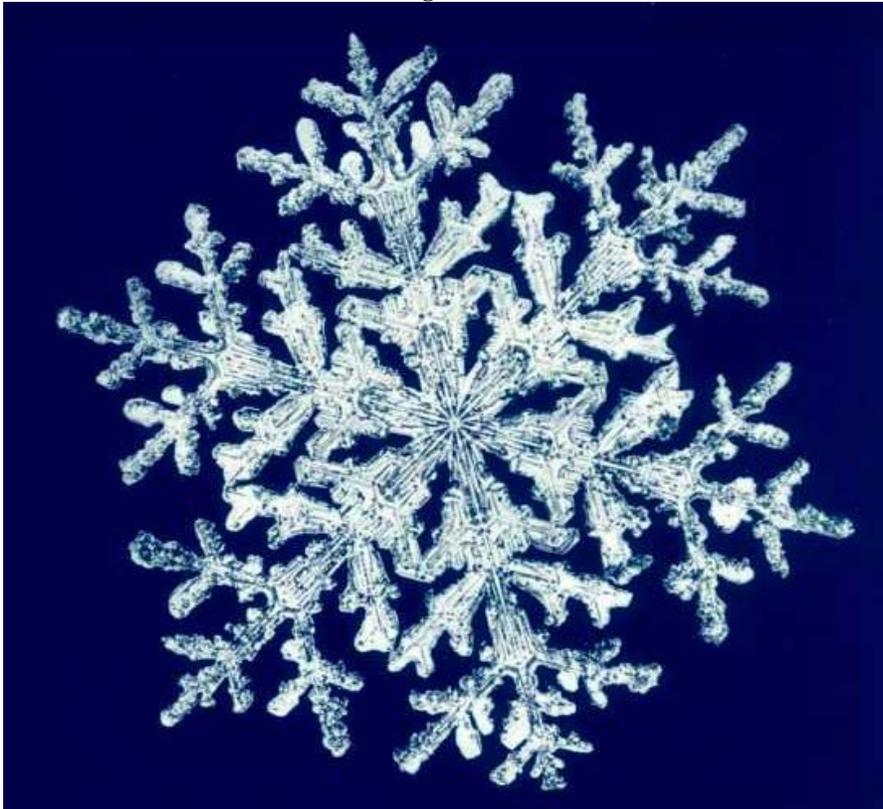
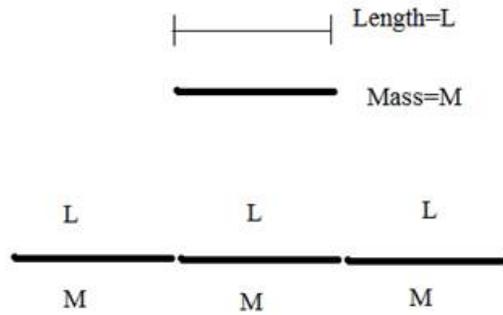


Image from: <http://www.ssec.wisc.edu/media/highlights2002/highlights2002short.htm>

## Non-Integer Dimension

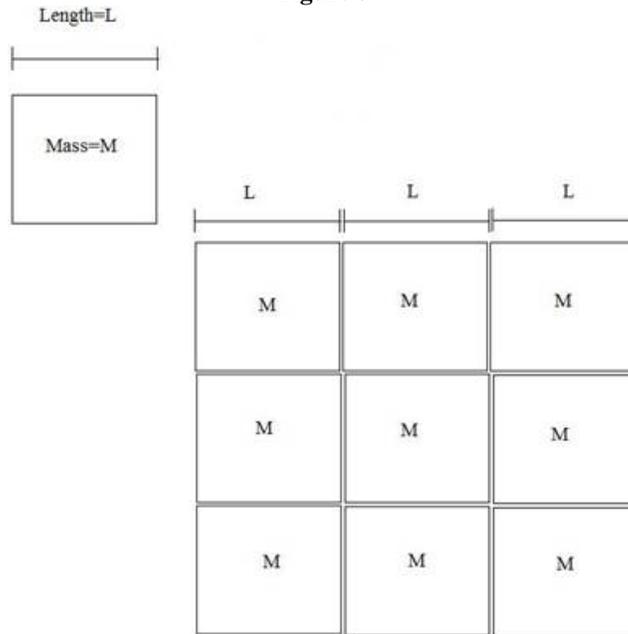
This characteristic may be hard to understand as we usually think of object in integer dimensions. The easiest way to describe a non-integer dimensional object is by the relationship of an object's mass to its length. The equation  $D = \ln(M) / \ln(L)$  expresses the relationship, where  $D$  is the dimension,  $M$  is the mass,  $L$  is the length and  $\ln$  is the natural log. Below are a few examples to describe this.

Figure 2



**Example 1:** In figure 2 above is a bar of length  $L$  and mass  $M$ . Below it is a larger bar drawn in such a way that you can see it is actually made up of three smaller bars of length  $L$  and mass  $M$ . Thus the larger bar has a length of  $3L$  and a mass of  $3M$ . Putting these values into the equation,  $D = \ln(3)/\ln(3)$ , it equals a dimension of 1 which is what we would expect from a bar.

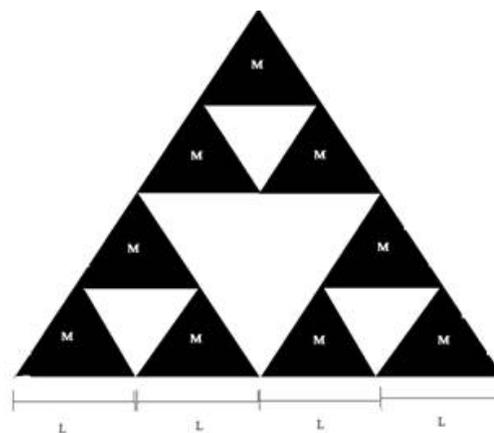
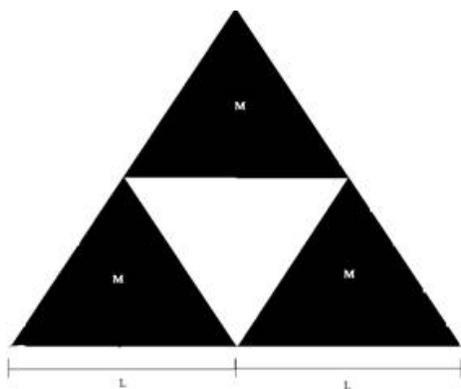
**Figure 3**



**Example 2:** In the upper left hand corner of figure 3 we can see a small square of length  $L$  and mass  $M$ . This time the box is tripled in length to make the larger square seen in the figure. The large square is also drawn to show it is made up of nine squares identical to the square in the upper left. From the diagram we can see the large square has a length of  $3L$  and a mass of  $9M$ . From the equation we can see that  $\ln(9)/\ln(3) = \ln(3^2)/\ln(3) = 2\ln(3)/\ln(3) = 2$  and we would expect a dimension of 2 from a square.

**Figure 4**

**Figure 5**



Example 3: In figures 4 and 5 are two Sierpiński's triangles, a fractal named after Waław Sierpiński which demonstrates a non-integer dimension. The fractal on the left has three smaller triangles all of length  $L$  and mass  $M$ , thus the fractal has a length of  $2L$  and a mass of  $3M$ . On the right, figure 5, the fractal has gone through another step and now has a length of  $4L$  and mass  $9M$ . Plugging these values into the equation,  $D = \ln(9)/\ln(4) = (2\ln(3))/2\ln(2) = \ln(3)/\ln(2) = 1.58$ . This time the relationship yields a non-integral dimension in between 1 and 2 dimensions. This is a similar dimension to the fractals I observed in my study.

### Cell Set-Up

The Copper fractals were grown in a cell, shown below (figure 6), made of two Petri dishes, copper wire, glue and the copper sulfate solution ( $\text{CuSO}_4$ ). Shown best in figure 7 and 8, two copper wires are used as electrodes for the cell. The outer electrode is made of copper wire shaped into a ring which is then placed along the inner edge of the outer Petri dish. The center electrode is copper wire poking through the inner Petri dish. This is done by first placing a dab of hot glue in the center of the inner Petri dish (to help seal and hold the center electrode), then the electrode is fed through the glue and the inner Petri dish while heated by a soldering iron (the tip of the electrode must have any glue or melted plastic removed with a razor). It is important to note that the center electrode must be poking through very little, such that the fractals have as close to a two dimensional growing space as possible. Once the electrodes have been made, the copper sulfate solution may be poured into the cell and the desired voltage can be applied.

Figure 6

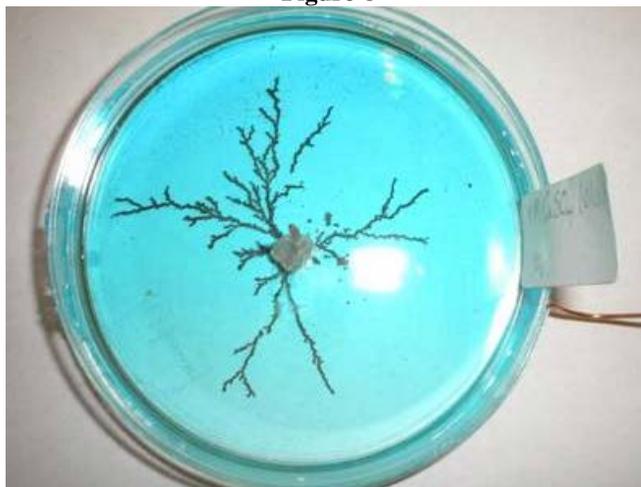


Figure 7

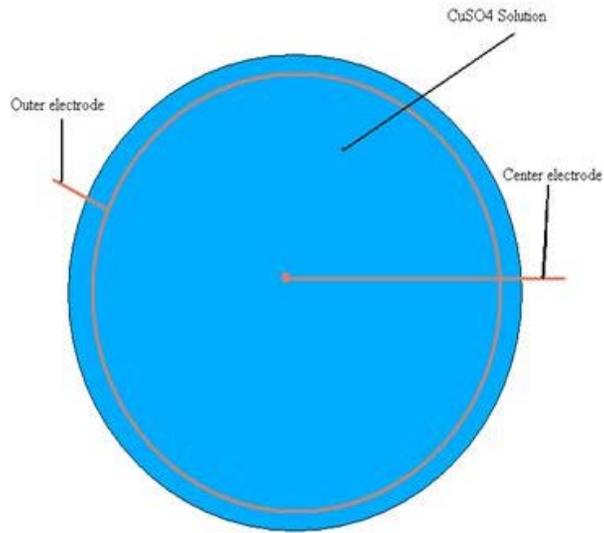
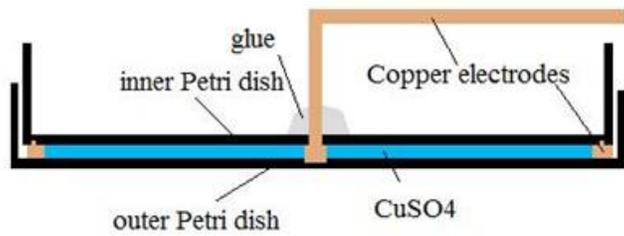


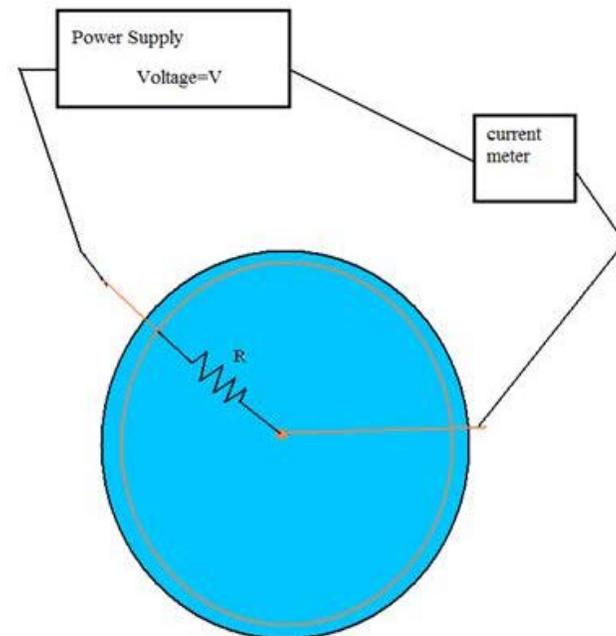
Figure 8



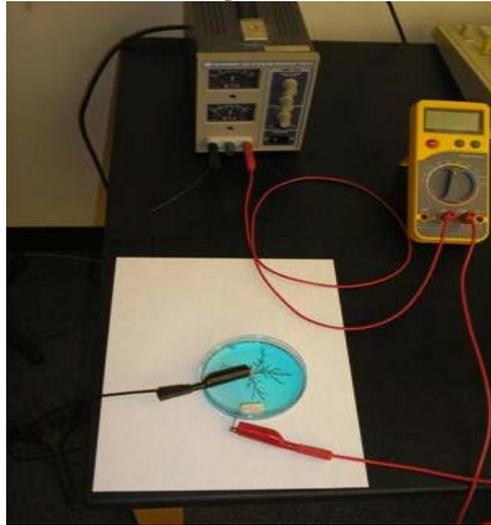
### Simple Circuit

A complete cell will have a voltage put across it by a power supply like the one shown below in figures 9 and 10. It is helpful to think of the power supply and cell as a simple circuit. A cell with a voltage across will cause the solution to act as a resistor. Though electrons aren't flowing from the center electrode to the outer, they are flowing to the copper in solution, causing a current. Therefore, the current of the circuit or rate of electron deposition, depends on the speed at which copper ions are able to take electrons; this process creates the resistance. This idea comes about when considering the time allowed for a copper fractal to grow, depending on the current and thus the effective resistance of the cell.

Figure 9



**Figure 10**



### **Copper Fractals**

The fractals themselves grow because the solution contains Copper  $2^+$  ions (any atom with a lack of or excess electrons). The copper ions “feel” the voltage and by way of electrostatic force, are driven towards the center electrode. While they move towards the center electrode, random motion due to other particles in the solution will cause the copper ions to have a sort of zig-zag path. Once an ion reaches the center electrode it will receive two electrons and crystallize. Because the movement of the ions is somewhat random, they will begin to form imperfections and as those imperfections grow they will become branches. A copper ion in solution will now have a much greater chance of running into the branch which is why the fractals do not form basic blobs. However, the amount of random motion is dependent on the voltage; the higher the voltage, the stronger the force driving the copper ions toward the center and the faster they will move, thus less time to have random motion.

### **Tests and Results**

Two main tests were done, testing the fractals dependence on concentration and voltage. When testing the concentration, it is ideal to see no change in dimension because high and low concentrations will respectively have more and less ions in solution. We expect the fractal dimension to depend only on the amount of random motion the ions participate in, which depends on how fast the ions move to the center electrode and thus depends on the voltage. Having more ions in solution will only cause the fractal to grow faster and not change the dimension. Along with testing the voltage three cells were made with varying concentration of sulfuric acid and one cell was made with a square outer electrode.

### **Concentration**

Above are the four fractals made with varying concentrations grown at 1.0 V and their respective dimensions. The first two fractals resemble each other well and are similar in dimension. The next two fractals are significantly less in dimension than the first two. The second two didn't grow as fast, which we expect, but they do resemble the beginnings of the first two fractals. Ideally we would expect the fractals grown at a concentration of 0.1 M and 0.05 M to grow the same fractal pattern as the 1.0 M and 0.5 M fractals.

1.0 M

0.5 M

0.1 M

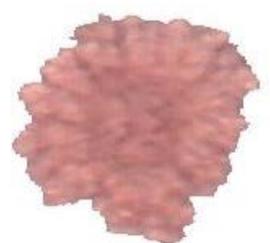
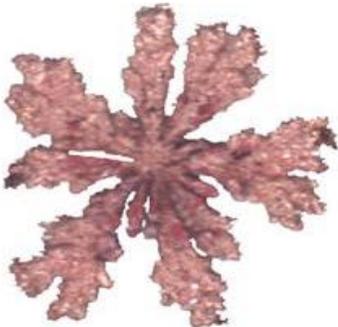
0.05 M

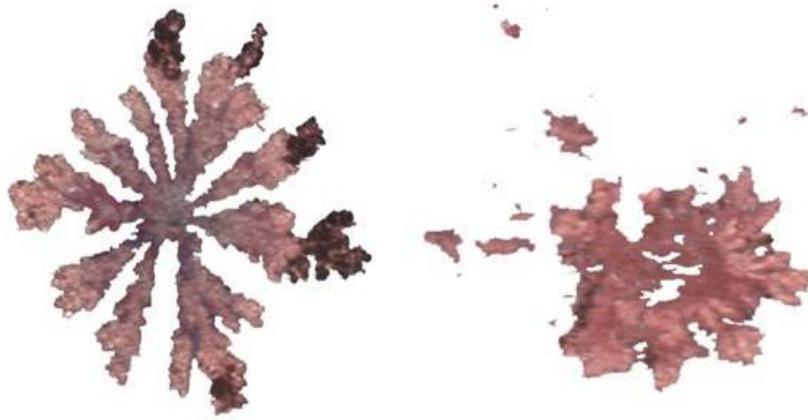
$D = 1.70$

$D = 1.72$

$D = 1.66$

$D = 1.65$





## Voltage

Below are the nine tests done with differing voltages, all grown at 1.0 M. The first four tests show desired results in that the increasing voltage increases the dimension. However, past 3.0 V the tests become very interesting and no longer grow in the expected pattern. At 3.5 V we can see the growth of a small thin branch and at higher voltages we observe more branches. This phenomenon is very interesting and not quite understood. It appears to be an instability, where at the right length of branch causes it to overtake the growth, but this does not explain why we see one branch on the lower voltages and many branches at high voltages. Although these fractals are not understood, we do observe an increasing dimension with voltage from the cell at 5.0 V to 10.0 V.

0.5 V

$D = 1.67$



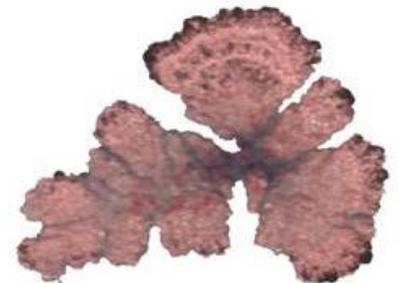
1.0 V

$D = 1.70$



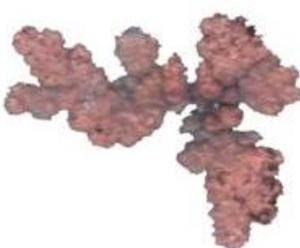
3.0 V

$D = 1.78$



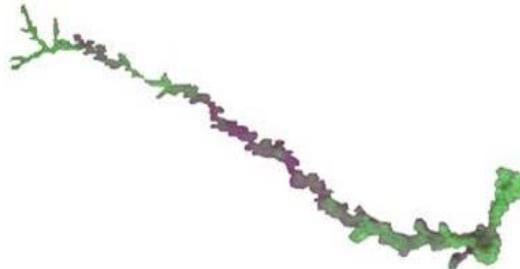
3.0 V

$D = 1.71$



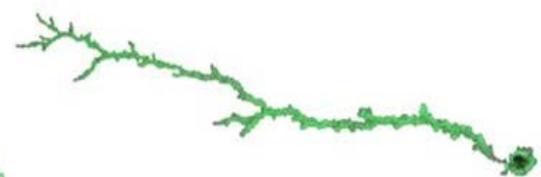
3.5 V

$D = 1.44$



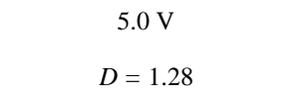
4.0 V

$D = 1.33$



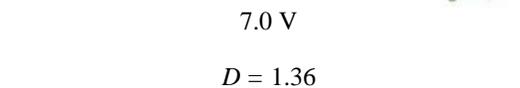
5.0 V

$D = 1.28$



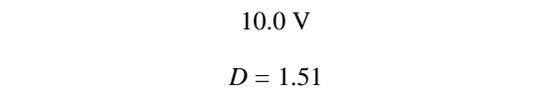
7.0 V

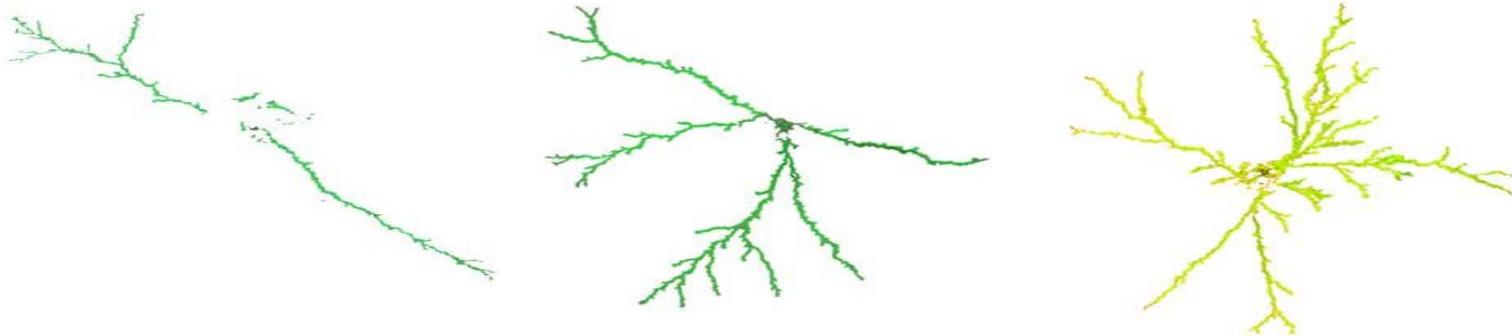
$D = 1.36$



10.0 V

$D = 1.51$

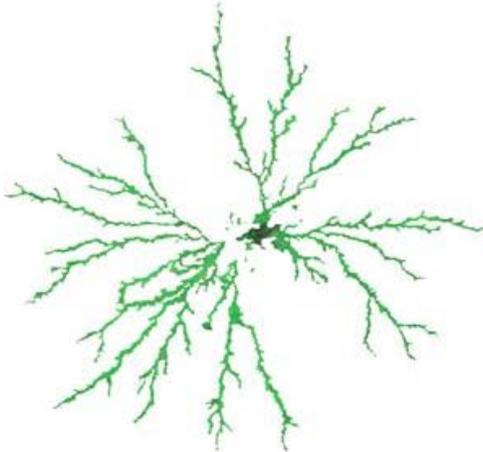




### Other Tests

The three tests done with varying concentrations of sulfuric acid were made up as follows: 0.5 M  $\text{CuSO}_4$  & 1M  $\text{H}_2\text{SO}_4$ , 0.75 M  $\text{CuSO}_4$  & 1M  $\text{H}_2\text{SO}_4$  and 0.75 M  $\text{CuSO}_4$  & 3M  $\text{H}_2\text{SO}_4$ . Their respective dimensions are 1.76, 1.74 and 1.69. The addition of sulfuric acid was done in an attempt to increase the current at lower concentrations, however the results were unexpected. The sulfuric acid only caused the fractals to grow very two dimensional and did not have a significant increase on the current. The final test was changing one cells outer electrode to a square shape, its fractal image is shown below (figure 11) and it had a dimension of 1.45. The square outer electrode produced a fractal similar to the 10.0 V fractal grown with a ring outer electrode, but there aren't conclusive results showing reproducibility to confirm no change in the fractal dimension.

Figure 11



### Future Plans

In further research I hope to show the fractals reproducibility and come to an understanding with why they grow these odd branches at voltages above 3.5 V. It would also be interesting to allow the lower concentration tests to grow out completely. The low cost and easy set up make this project simple enough to carry on with little time needed.