Finding Latitude from a Tabletop Foucault Pendulum

Nicole Tennessen and Nathan Wichman

Advisor Derin Sherman

It is currently common knowledge that the Earth rotates. However, several hundred years ago, this was not the case. In 1851, Leon Foucault provided a simple example of Earth's rotation: the Foucault Pendulum. The most famous of his pendulums hung in the Pantheon and was a celebrated 67 meters long (Foucault Pendulum, 2013). The great length was necessary to avoid unwanted elliptical motion. Ideally, Foucault Pendulums swing in only one plane. Ellipsoidal motion is caused when the velocity is not pointed completely within this plane of swing for the pendulum, meaning there is a component to the velocity that is perpendicular to this plane. Long pendulums naturally avoid this elliptical motion. Over time, the plane of swing rotates (a phenomena known as precession) which was proof the Earth rotates.

Depending on the pendulum's latitude (where the pendulum is located on Earth in reference to the equator), the plane where the pendulum swings will precess at different rates. This rate is determined by the amount of time it takes to precess 360 degrees. Therefore, a pendulum located at either of Earth's poles will result in the pendulum precessing three hundred and sixty degrees in the time of one day. A pendulum located at the equator will stay in one plane of motion and never precess. The equation derived in many textbooks that relates the Earth's rotational rate to the precessional rate at a given latitude is

$$\Omega_F = \Omega_{Earth} \sin \theta_{latitude}$$

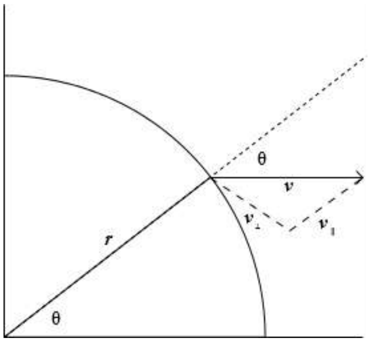

Figure 1: Rotational Reference Frame

where Ω_{Earth} is the rotational rate of the Earth, $\theta_{latitude}$ is the latitude of the pendulum, and Ω_F is the rate at which the pendulum precesses at the given latitude (taken from Pittsburgh article). There is a simple mechanics explanation for this equation. Figure 1 displays the rotation of Earth about the vertical axis. Outside of this rotational reference frame, the point where the radius $\boldsymbol{r}$ (measured from the center of the Earth) intersects the Earth's crust moves at a velocity $\boldsymbol{v}$, which is related to Ω_{Earth}, the rotational rate of Earth. $\boldsymbol{v}_\perp$ is the component of the velocity $\boldsymbol{v}$ perpendicular to radius $\boldsymbol{r}$: this is the component of the velocity $\boldsymbol{v}$ that creates rotation about radius $\boldsymbol{r}$ within the rotating reference frame. This is the rate we are interested in as Ω_F is related to $\boldsymbol{v}_\perp$ in the same way Ω_{Earth} is related to $\boldsymbol{v}$. Therefore, we can treat Ω_F as if it were equal to $\boldsymbol{v}_\perp$ and Ω_{Earth} equal to $\boldsymbol{v}$. To find how $\boldsymbol{v}_\perp$ is related to $\boldsymbol{v}$, trigonometric properties are employed: $\boldsymbol{v}_\perp$ and $\boldsymbol{v}_\parallel$ ($\boldsymbol{v}_\parallel$ is the component of the velocity $\boldsymbol{v}$ that is parallel to radius $\boldsymbol{r}$) are perpendicular by definition; to find $\boldsymbol{v}_\perp$, we simply multiply by the sine of the angle between radius $\boldsymbol{r}$ and velocity $\boldsymbol{v}$. This angle is the latitude measured from the equator. We can see that when the pendulum is located at the pole (the vertical axis, $\theta = 90$ degrees), $\boldsymbol{v}_\perp$ will equal $\boldsymbol{v}$. The precessional pendulum rate Ω_F will equal the precessional rate of the Earth Ω_{Earth}. In contrast, when the pendulum is located at the equator (the horizontal axis, $\theta = 0$ degrees), $\boldsymbol{v}_\parallel$ will equal $\boldsymbol{v}$ meaning $\boldsymbol{v}_\perp$ will equal 0; pendulum will not precess. Figure 1, which explains the relationship between Earth's rotational reference frame motion and the rotational rate experienced at a given latitude, effectively models the equation above.

This discussion begs the question whether a portable Foucault pendulum can be built to accurately determine the latitude of a pendulum on Earth's surface at different latitudes. The first step in determining the answer requires the building of a smaller pendulum. While using a larger pendulum—on the order of 20 m—has certain advantages (e.g., taking a longer time for air resistance to noticeably damp the pendulum and naturally damping out unwanted ellipsoidal motion), building a smaller pendulum is more beneficial because it allows for easier transportation and requires less space to set up. However, if the energy the pendulum loses each swing is replaced, the damping would no longer be an issue. The replacement of the energy would allow the pendulum to swing for an extended period of time. Our design drove the pendulum using magnetic fields produced by electromagnets that repel a magnet attached to the bottom of the pendulum bob. With each pass of the bob over the electromagnet, the energy lost is replaced by a repulsive magnetic force from the electromagnet. The second issue of reducing the ellipsoidal motion present in a shorter pendulum is discussed in an article written by R. Schumacher and B. Tarbet (2009). Our set up modeled theirs: we employed the use of an aluminum disk that damped the ellipsoidal motion using eddy currents. The disk reduced the amount of motion perpendicular to plane of swing without affecting the motion within the plane of swing. Therefore, the pendulum stayed within a single plane of motion. Also, repelling the bob each time it passes over the electromagnet versus attracting the bob as it swings in lessens the unwanted ellipsoidal motion.

Our experimental set up is seen in Figure 2. The pendulum is attached to the ceiling using magnets and an aluminum support. The length of the string is attached to both the aluminum support and the bob with a pin vise. The string is monofilament fishing line, Essentials South Bend test rated at 25 lbs. An advantage to using the fishing line is the line has no memory and should not have a preferential swing. However, the line did stretch when the bob was attached and therefore needed to be adjusted occasionally. Once the line stretched to its maximum length, it did not need to be adjusted. Attached to the bottom of the string is the bob. The bob was a .82 kg brass cylinder that was 2 inches in diameter. A neodymium-iron-boron magnet (1 inch diameter, 3/8 inch tall) was secured to the bottom of the bob. The magnet was concentric with the string. Centered below the pendulum at the equilibrium position, the electromagnetic coil is concentric with the aluminum damping disk, magnet, and string. The coil was constructed using 90 meters of 30 gauge double stranded wire, had a measured resistance of 29 ohms, 450 turns, and an average radius of 1.38 inches. The aluminum disk has in inner radius of 8.27 inches and an outer radius of 16.53 inches. When the pendulum is put into motion, the pendulum swings across the electromagnetic coil, triggering the timing circuit discussed in the next section. The circuit turns on an electromagnet and repels the magnet attached to the bob. The electromagnet turns off after a brief time; the pendulum finishes swinging to the full amplitude. The pendulum reverses direction and the process repeats.

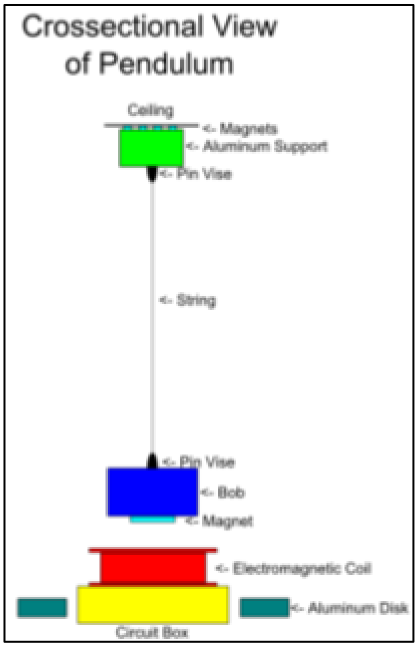

Figure 3: Diagram of the Pendulum Apparatus

The circuit turning the electromagnet on and off is seen in Figure 3. All three timers used are LM555s; the capacitors are ceramic; the mosfet is an IRFZ10. The coil of wire seen in Figure 2 is composed of wrapping double stranded 30 gauge wire. One strand of wire is used to sense the presence of the magnet attached to the bob (this will be referenced as the "sense coil") and the second coil is turned on and off to drive the magnet (this will be referenced at the "drive coil"). By running current through the drive coil, a magnetic field is created that repels the magnet on the bob. This is used to replace the energy lost each swing. When the pendulum swings over the sense coil, there is a voltage drop across the coil. The signal as the magnet passes over the sense coil has a shape similar to a sine wave with an amplitude of 100 mV. This signal can be seen in Figure 4 on the screen of the oscilloscope. The bottom signal is the signal coming into the circuit from the sense coil. Depending which way the magnet is oriented on the bob (dependent on which side of the magnet is facing the electromagnet), the voltage will first dip below the axis and secondly rise above the axis or first rise above the axis and secondly dip below the axis. Our circuit is designed to trigger when the orientation of the magnet produces a negative voltage followed by a positive voltage. Since the original signal is only -100 mV, the voltage must then be amplified. In order to trigger logic gates and timing components used later in the circuit, the voltage must be brought up to 5 volts.

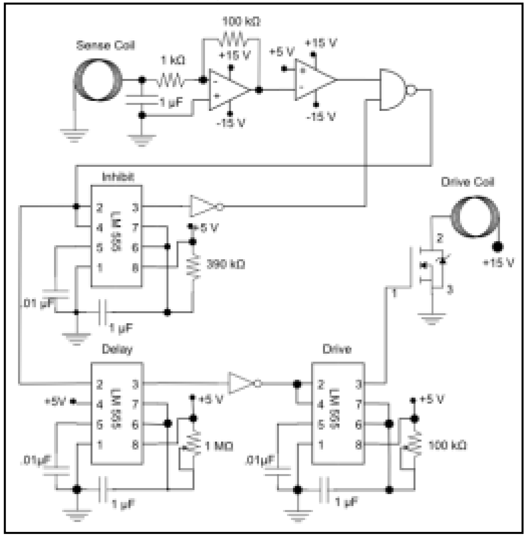

Figure 2: Schematic of the Timing Circuit

Before amplifying the signal, low-pass filter is used to reduce the noise from the original signal. To amplify the voltage, an operational amplifier (LF412) is used. The gain of this amplifier is 100, bringing the voltage to a level that is high enough to reach

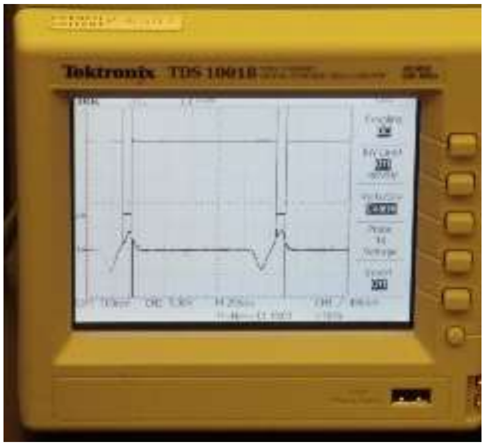

Figure 4: Signals within Circuit. Top Signal - Output Signal. Bottom Signal - Input Signal.

the threshold. Once the input signal is amplified, this signal is then run through a comparator (LM311) that turns the analog signal into a digital signal. This is sent to the NAND gate (74HC00). The other input to the NAND gate is attached to the inhibit timer. As long as the inhibit timer is low, the signal from the comparator will be able to pass through the NAND gate. When the signal passes through the gate, two timers are started simultaneously.

The first timer is the inhibit timer. While the timing of the pendulum should only produce one signal being processed at one time, this timer guarantees only one signal from the pendulum swinging over the sense coil to pass through and turn on the electromagnetic coil. Once the signal has reached the inhibit timer, the timer outputs a low signal which is inverted by the inverter into a high signal. This is what prevents the NAND gate from firing again. If another signal were to reach the NAND gate before the inhibit timer has wound down, the inhibit signal would still be high and the NAND gate would not allow the second signal to proceed. Once the inhibit timer has finished, the timer puts out a high signal which is inverted again by the inverter into a low signal. This allows the NAND gate to fire when another signal from the pendulum passing over the sense coil reaches it. Another reason the inhibit timer is necessary is because it prevents the NAND gate from firing again when the drive coil is turned on. The drive coil produces a change in voltage across the sense coil that is large enough to trigger the circuit again. The change in voltage can be seen as sharp peaks within the sense signal. This is a problem because the drive signal would continuously trigger itself. We want the only the magnet on the bob to trigger the drive coil on and off. The inhibit timer guarantees this.

The signal from the NAND gate also triggers another timer called the delay timer. This timer delays the signal to turn on the electromagnetic coil until the pendulum is in to correct position in the swing to receive a repulsive push. This time is can be adjusted by the potentiometer attached to the delay timing chip. Once the delay timer has wound down, it passes the signal to the drive timer. This chip determines how long the electromagnet is on. Again, the potentiometer attached to the drive timer allows for this time to be adjusted. When the drive timer receives a high signal from the delay timer, the electromagnetic coil is turned on, creating a repulsive force on the magnet attached to the bob. After the driver timer has wound down, the electromagnet turns off. One end of the coil is held at 15 volts. The coil is turned on when the other end of the coil (the part connected to the output of the drive timer) is driven to ground. Therefore, when the coil is off, it is held at 15 volts and when it is turned on, one end of the coil is driven to ground. This behavior produces a square wave signal which can be seen in Figure 4. The top signal is the drive coil: when it is off, the signal is 15 volts and the dip to ground when it is turned on.

During the designing of the circuit, we used a very small coil (R = 4 ohms) and a very small pendulum (< 0.5 m) to determine what sort of signal the bob passing over the sensing coil produced and to gauge a general timeline of when the timing chips needed to trigger. Once the original circuit worked, we adjusted out design for a larger pendulum. The string was lengthened to just over 2 meters and a coil with higher resistance was needed to prevent too much current from flowing. A larger coil also will produce a larger magnetic field. This means that a lower signal can be sensed by the sensing coil and when the drive coil is turned on, it produces a stronger repulsive force.

Since the circuit and pendulum length were adjusted according to the new pendulum, we were now ready to place the electromagnetic coils and the aluminum damping disk underneath the pendulum. The first question was where to place them relative to the pendulum and the second question was where to place them relative to one another. Placing the coils so the bob passes very close to them results in a stronger input signal through the sense coil and a stronger repulsive push. Raising the magnet so it is higher away from the coils results in a lower signal and lower driving force. Lowering and raising the damping disk also affects how much the bob's amplitude is damped. If it is closer to the bob, the damping effect is greater. The final stages of the research were spent making minor adjustments to the relative positions to the pendulum.

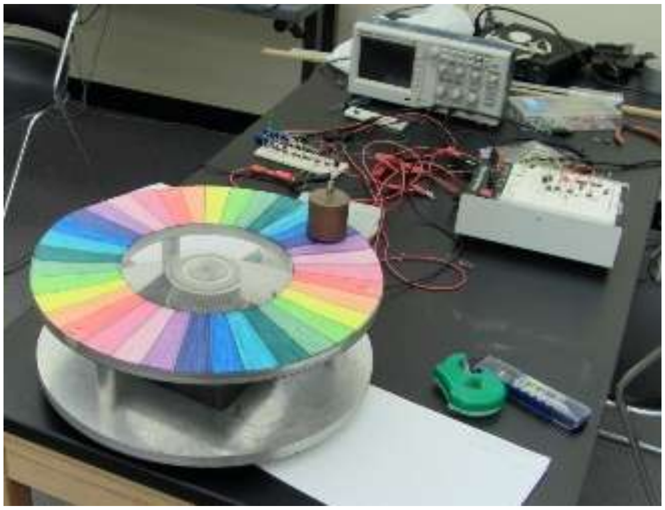

Figure 5: Foucault Pendulum

The latitude of the pendulum was roughly 40 degrees. This correlates to a precessional rate of about 10 degrees per hour. In order to notice this rate, we drew a color wheel with 10 degree increments. Therefore, if we started the pendulum swinging in one plane, the change in one hour will be noticeable. However, when the pendulum started swinging, we did not observe the correct precessional rate. Occasionally, the rate was double the expected rate, others it seemed to move slower. When the pendulum was left for long periods of time, the pendulum was not aligned with where the plane should have been (given the correct rate). The pendulum was locked into on plane of motion: we would observe no more rotation of the plane of swing. Each time the pendulum locked into a plane of motion, we would stopped the motion and attempt to correct the issue causing an incorrect precessional rate and restart the pendulum swinging. Ultimately, we believe that the inconsistency with the theoretical precessional rate is due to asymmetries in the alignment of the pendulum. Many individual pieces must be lined up very accurately to ensure the proper measurements are collected. The inner radius of the aluminum disk must be concentric with the electromagnetic coils, magnet attached to the bottom of the bob, as well as the pendulum string. The coil and aluminum disk seemed to work together best when the top of the coil is flush with the aluminum disk. Along with being flush, the top of the coil and aluminum disk most like need to be level. Overall, many components are dependent on being aligned correctly. The next step is ensuring each component is correct positioned.

To aid in the correct alignment of the element, concentric circles where printed. Two sets of circles were used: one set to align the magnet on the bob and the other to align the aluminum disk with the electromagnetic coil. The first set was printed on regular paper. The outer circle had a radius equal to the radius of the bob and the inner circle had a radius equal to the magnet attached to the bottom of the bob. Both circles shared a common center point. This printout was used to place the magnet on the bob. The second set of circles were printed on a transparency: this allowed us to see more easily line the coil with the aluminum disk. While these circles aided in aligning pieces more closely to one another, we still did not have a mechanism for aligning the magnet and bob with the electromagnetic coil and aluminum disk. This was attempted by aligning the two sets by hand.

Bibliography

Foucault pendulum. (2013). In Encyclopædia Britannica. Retrieved from http://www.britannica.com/EBchecked/topic/214686/Foucault-pendulum

Schumacher, R. A., & Tarbet, B. (2009). *A short foucault pendulum free of ellipsoidal precession*. (Master's thesis, Carnegie Mellon University) Retrieved from http://arxiv.org/pdf/0902.1829v2.pdf