Various Methods of Experimentally Measuring the Radius of Atoms; the Ramsauer-Townsend Effect and Electron Diffraction

Introduction

It's commonly known that particles act like waves, and this introduces interesting quantum mechanical effects between interacting particles. This quantum behavior allows for an experimental determination of various atomic properties. Two examples of this behavior are seen in the Ramsauer-Townsend Effect, where the cross section dips to an extreme minimum, and electron diffraction, where the electron will diffract off an atom like a wave. Both can be used to find the radius of their respective scattering atoms. I will observe both in order to try and experimentally determine the radius of Xenon with the Ramsauer Townsend effect, and the geometry of Aluminum with diffraction.

I. Ramsauer-Townsend Effect

A. Background Information

The Ramsauer-Townsend effect is an observable effect where the cross section of a xenon atom drops to almost zero, before increasing again as the electron energy is increased. At the low energies where this is seen, the collisions are all elastic, and classically the elastic cross section should decrease as electron energy is increased. Instead we see the dip previously mentioned. This is due to the electron acting like a wave, and we can equate the atom to a potential square well. At certain wavelengths of the electron, the cross section will go to zero. To understand this, we need to develop a general background for the effect.

1. Electron Wavelength and Potential Square Well

For the Ramsauer-Townsend effect, we predict the electrons to act like waves. The wavelength of the particle as determined by De Broglie is:

$$\lambda = \frac{h}{p}$$

This can then be related to the momentum of a particle in a potential as defined by the Schrodinger equation:

$$p = \hbar k = h/\lambda$$

Then there is the kinetic energy of a particle as related to momentum:

$$E = \frac{p^2}{2m}$$

Which shows that for a specific energy of a particle, there is a corresponding wavelength; that particle in this case being an electron. As previously stated, the Ramsauer-Townsend effect is the result of the electron acting like a wave, while the Xenon atom acts as a potential square well with the following boundaries:

$$V(x) = \begin{cases} 0 & x < -a \\ -V_o & -a < x < a \\ 0 & x > a \end{cases}$$

Here a can be seen to be the radius of the atom. The incident wave will interact with the walls of the well and either partially reflect or transmit. At $x = -a \; and \; x = a$ the wave changes due to the difference in potential. The effect depends on the length of the well, its potential depth, and the incident wavelength. What we want to find for there to be apparently zero cross section is a situation in which the wavelength is transmitted fully, without any reflection.

As the electron increases in energy, the wavelength decreases. This change in wavelength is the cause of the Ramsauer-Townsend effect. What happens with this one-dimensional square well is that when the wavelength of the incoming electron is twice as long as the width of the well, then the wave travels through without any reflection. The transmission coefficient for the equation is one in this situation. Physically, we can say that the wavelength enters the well, and reflects and transmit off the x=a potential wall. However, when the wavelength travels half a wavelength back to the x=0 potential wall, it will constructively interfere with the new incoming wave, and thus the entire wave will be transmitted.

As we change the wavelength of the electron, it will at one point be approximately twice the well width, and that is where we see the drop in cross section. Experimentally, we can find the energy associated with this dip, and using this equation can find the wavelength associated, and then the width of the well that we assume is indicative of the diameter of the atom.

$$E = \frac{h^2}{2m\lambda^2} = \frac{h^2}{32mr^2}$$

(Equation 1)

2. Cross Section

Now that we understand why the Ramsauer effect occurs, I had to also understand how to calculate the cross section from the data that I gathered. In this experiment, I thermionically emitted electrons through a vacuum tube that houses Xenon. There are various electrical connections on the Thyratron that operate the electron energy, and also for voltage and current readout. Therefore, I had to find a relation between the cross section and current. Based on the paper by Kukolich[1] I developed the following understanding.

We can see the electrons going through the Xenon gas as an attenuating beam over a distance. Since the size of the tube is known, and the system is very clean, we can write down this equation relating the flux at the collecting cathode plate J_p, and the initial flux J_o:

$$J_p = J_o(1 - P_s)$$

P_s is the probability of scattering, which makes sense as the flux should decrease depending on the scattering probability. Now, we can't measure the flux of the system, and we don't have to. Instead, we can easily equate the plate flux and the initial flux to plate current I_p and shield current I_s (current due to the scattered electrons). We then arrive at this equation:

$$I_p = I_s f(V)(1 - P_s)$$

$f(V)$ is a geometric factor that includes the ratio between the angle of interception at the plate and that of the shield's, and it also includes factors due to the space charge effects near the cathode. The space charge effect is a product of thermionically emitted electrons and affects the emission of the electrons. Now, we can't really calculate this. The solution that Kukolich[1] used was to freeze out the Xenon with liquid nitrogen, reducing the probability of scattering to zero. This removes all the Xenon from the gas phase, and the electron beam travels through the scattering zone unhindered. Since there can't be any scattering, the currents I see will be almost entirely due to $f(V)$, and we can make the following approximation:

$$f(V) \approx {I_p}^*/{I_s}^*$$

Now that I can experimentally find the geometric factor, I can then find the cross section. We have a relationship between the probability of scattering and the cross section here:

$$P_s = 1 - e^{-l(N\sigma)}$$

After substituting and solving the equation for the cross section, we arrive at the next equation:

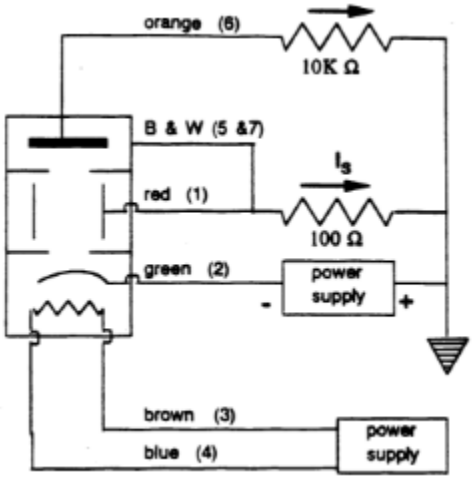
$$\sigma = \frac{1}{Nl}\ln\left(\frac{I_s\,{I_p}^*}{{I_s}^* I_p}\right)$$

I can now calculate the cross section depending on the four currents that I read out. It must be said that this calculation isn't the best or the most accurate. Due to the shield currents being very similar and really only contributing to noise, in my final analysis I only considered the plate currents.

$$\sigma = \frac{1}{Nl}\ln\left(\frac{{I_p}^*}{I_p}\right)$$

(Equation 2)

B. Lap Setup for Ramsauer-Townsend Effect



The above diagram shows the circuit that I built around the Thyratron. This setup was proposed by Kukolich specifically for this experiment. Before I get into the circuit, I'll explain how the Thyration itself works. It emits electrons thermionically by heating up the filament with a voltage. Then, a potential difference is created between the cathode and the anode, which accelerates the electrons from the cathode to the anode. Xenon gas is present throughout the tube, and as the electrons travel through a little metal box (the shield), they collide with the Xenon atoms and scatter. The shield collects all of the scattered electrons, while the anode plate collects all the electrons that didn't collide. At the low energies I had in the experiment, in the range of a few electron volts, all of the collisions are going to be elastic. With that in mind, I built this circuit which was used previously in other labs for this experiment.

Connections 3 and 4 at the bottom are for the heater, where I applied a 5 volt potential. In the schematics, the heater is recommended to be run at 6.3 volts; various papers however recommended running the Thyratron at lower voltages to decrease space charge effects, which would reduce noise. Another voltage is applied on the electron cathode (connection 2) which is negative in relation to the shield and anode (plate) voltage. The shield voltage (connections 1, 5, and 7) are connected to a 100 Ω resistor, and the current and voltage is read across the resistor. Finally, the collecting plate is connected to a 10K Ω resistor, which the current and voltage is also read across. The circuitry was all placed inside a circuit box, with connections for power and measurements. On the box I mounted the Thyratron. I used an electrometer to measure the two currents, and a digital multimeter to accurately determine voltage potential across the cathode.

With the measurements, I went from 0 to 10 volts, increasing in steps of 0.05 volts between 0 and 5 where the Ramsauer-Townsend effect is seen. Beyond 5 volts, I increased the difference between measurements, as those weren't as important or entirely necessary for the experiment.

When that was completed, I dipped the Thyratron tube into liquid nitrogen, and made measurements. Ideally, the liquid nitrogen would be in a Dewer flask and the circuit box would be placed on the flask with the Thyratron tube being completely submerged. However, this wasn't the case for me. I had to put the liquid nitrogen in a Styrofoam cup, which was supported by a beaker. I assume that my setup caused the liquid nitrogen to evaporate faster, and meant I had to add nitrogen to the cup more often to maintain freezing temperatures.

C. Collected Data

With the Ramsauer-Townsend effect, the data came out exceptionally clean. This was to be expected, as there wasn't really very much that could have dirtied the data as the only interacting particles are electrons and Xenon, both of which are extremely stable. For the free Xenon, I saw exactly what I had hoped for. The plate current would increase to a certain level, and then begin decreasing as we can see in Figure 1. When I froze the Xenon, I got a mostly linear growth in the plate current. However,

there is a slight curve to the line, which affected the final cross-sectional analysis.

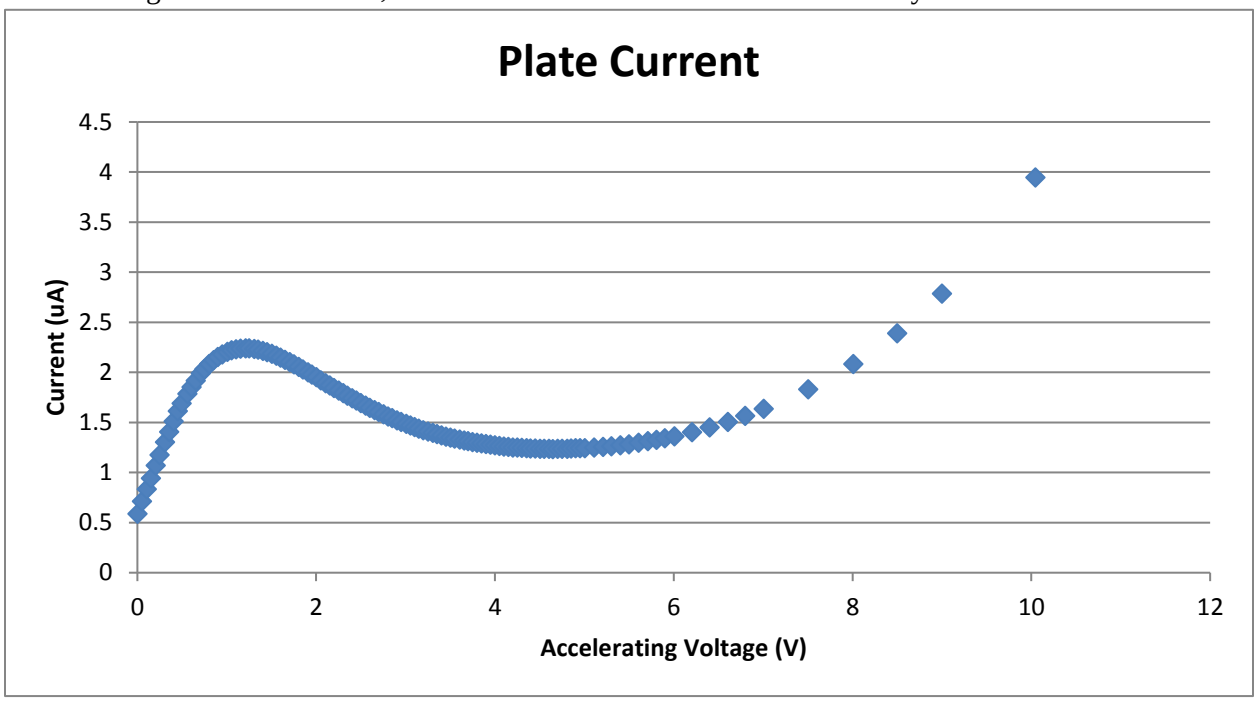


Figure 1: Plot of plate current against the accelerating voltage for when the Xenon is gaseous and free to interact with the electrons

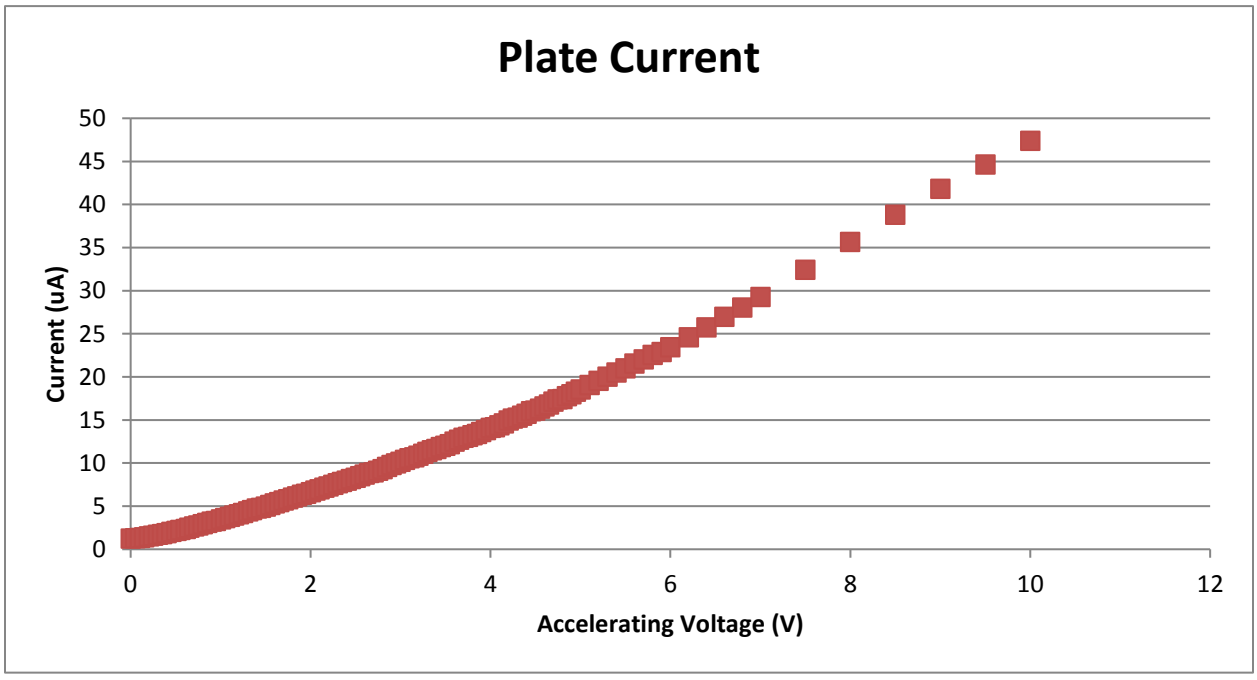


Figure 2: Plot of plate current against accelerating voltage for when the Xenon is frozen out, and the scattering of electrons should be zero

Taking Equation 2 that relates the cross section to the plate currents, I used the values I acquired to find the cross section at increasing electron energies. The resulting graph was exactly what I had

expected, where there is a dip in the cross section in the ballpark of 1 eV. However, my initial minimum was at approximately 0.425 eV, which is 67.5% off from the accepted value of 1 eV. While these measured values for the cross section are different than previously conducted experiments, it is however indicative of the quantum nature of particles.

Then, using the 0.425 eV minimum I measured in the equation derived from the 1 dimensional potential well for the radius, I found the Xenon radius to be 4.13Å. This is close the actual radius of Xenon, 1.08Å, but I also have to take into account the lower than expected eV. With 1 eV, the calculated value is around 10 Å, which while close is an order of magnitude off.

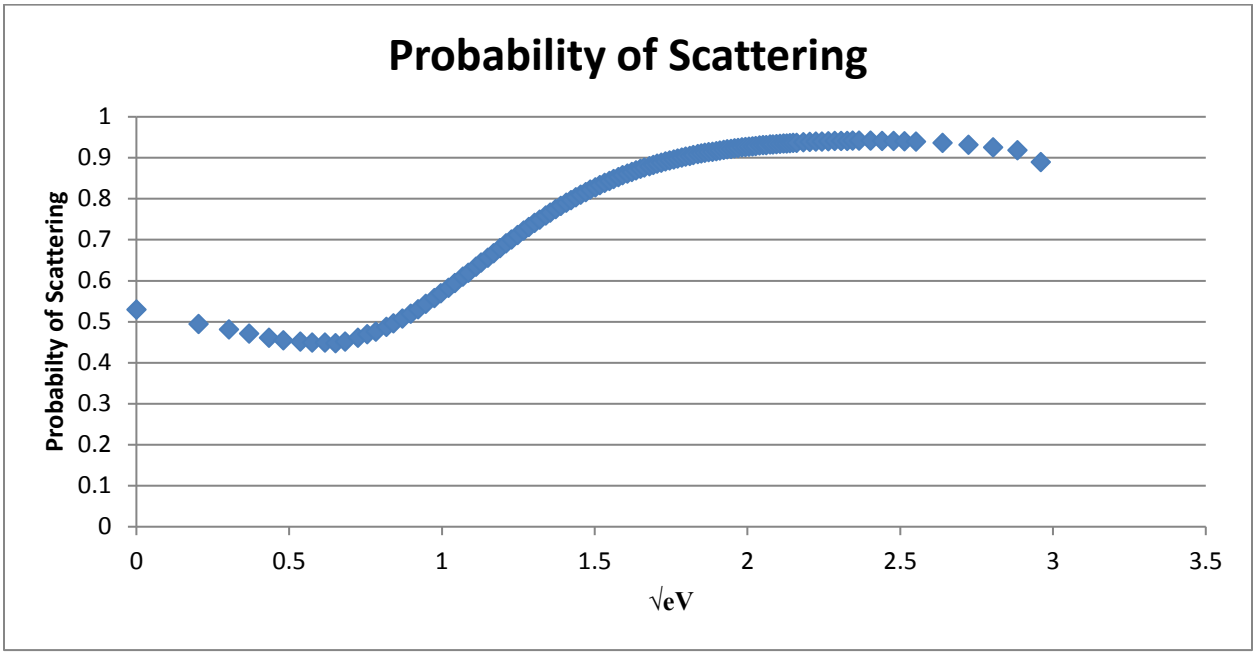


Figure 3: Plot of the probability of scattering against the square root of electron voltage ($\sqrt{V - V_s}$)

D. 3D Spherically Symmetrical Potential Well

Now that I had the data, it became increasingly apparent that a 1D representation of the potential well isn't very accurate. Instead, scattering off a 3D spherically symmetrical potential well is required for more accurate calculations.[1] This first means putting the 3 dimensional Schrodinger equations into spherical coordinates. Since the waves are also 3 dimensional, I used partial wave analysis, which decomposes the wave-function into its individual angular momentum components. With the wave-function dependent on the angular momentum, it becomes much easier to solve the equation, as the experiment relies on low energy electrons. The wave will be at a significantly low energy where the angular momentum can be assumed to be zero, meaning I only have to solve for a single wavefunction.[1] Also, since the well is spherically symmetrical, we don't have to worry about the θ or ϕ components of the Schrodinger equation[2]. It leaves only the r variable, which is called the Radial Equation[2]:

$$\left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{l(l+1)}{r^2} + k^2\right] R_{El}(r) = 0$$

We can see that the Radial Equation resembles a Bessel differential function, and that the solutions would be combinations of Bessel functions. A 3D potential well has limits like so:

$$V(r) = \begin{cases} 0 & if\ r > a \\ V_o & if\ r < a \end{cases}$$

Then the solution for the area inside the well is:

$$R = Aj_l(k'r)$$

$$k' = \sqrt{\frac{2m_o(E + V_o)}{\hbar^2}}$$

Looking outside of the well, the solution is:

$$R = Bj_l(kr) + Cn_l(kr)$$

$$k = \sqrt{\frac{2m_o(E)}{\hbar^2}}$$

Here, j_l is a Bessel function, and n_l is a Neumann function. We can equate them with a logarithmic derivative, which can be used to solve for the phase shift of the potential well. I want to have an equation for the phase shift, as when the shift is at nπ, the partial wave is completely transmitted. Since we are looking at partial waves at $l = 0$, I only have to solve for one Bessel function. After working through the problem, I found the following relationship where the wave is completely transmitted.

$$\frac{k'}{k} = \frac{\tan(k'a)}{\tan(ka)}$$

(Equation 3)

Through S-wave scattering, there is also another relationship found. We assume that outside of the potential well the value for ka is going to be very small[3]; which means $\tan(ka) = ka$. Plugging this back into Equation 3, we arrive at the following relationship.

$$\tan(k'a) = k'a$$

(Equation 4)

With Equation 4, I can calculate the angular wave number k', and then plot and solve for the value of a. This value will be the experimentally derived radius of the atom.

Systematic Error in the Thyratron

The Thyratron, though a very simple device, does possess some issues that affect the accelerating voltage for the electrons. There are two that must be included in my final analysis, and these where both discussed by Woolsey[4] in his paper. He described a method to test for both, which involved freezing out

the Xenon, reversing the accelerating potential and measuring the shield current until with increasing potential.[4] The current should at some point drop to zero, as this retarding potential should prevent the electrons from hitting the shield.

1. Contact Potential

In the Thyratron, the shield and plate cathode are made of two different substances; the shield plate is nickel, while the cathode has a barium oxide coating.[4] The nickel of the shield has a higher work function than the barium oxide coating.[4] Contact is made between the shield and the plate via the gas, meaning a flow of electrons occurs when they are at the same temperature and there is no potential difference applied between the two.[4] This means that a negative potential exists in order to balance the flow of electrons, so we must include this contact potential into our final voltage.

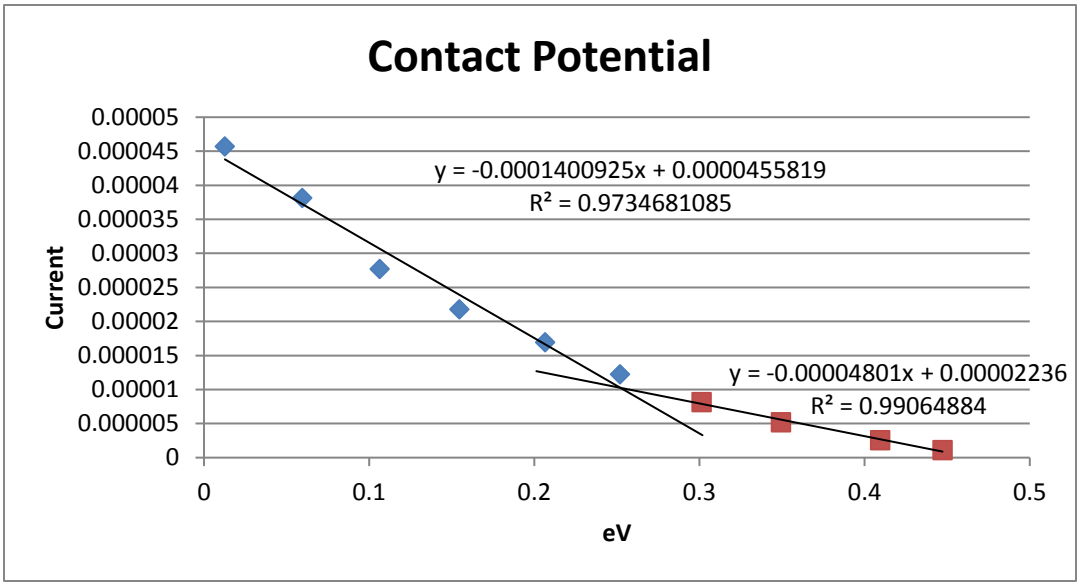


Figure 4: Plot of the log of the current against the electron voltage (V-Vs). Two lines of best fit were found.

With how Woolsey[4] laid out the experiment, we plot the log of the current against the electron voltage, and there should be two distinct linear slopes in the plot. We can then find the lines of best fit, and then the intersecting point is the contact potential. For my set up, the contact potential was 0.25 volts.

2. Initial Thermionic Electron Energy

Since the Thyratron is thermionically emitting the electrons, it's apparent that there must be an initial energy for each electron before acceleration. For thermionic electrons, the energy distribution is close to Maxwellian[4]. So, if the accelerating potential is reversed in my setup, it essentially becomes a retarding potential. Woolsey[4] shows that if electrons are collected by an electrode at a retarding potential V_p, the relationship with the shield current is,

$$I = I_o e^{\left(\frac{3V_p}{2V_e}\right)}$$

Where V_e is the mean thermionic emission energy of the electrons. We can then plot the shield current against the retarding potential, and the slope of the line allows me to find V_e. For my setup, the calculated value was 0.5 volts.

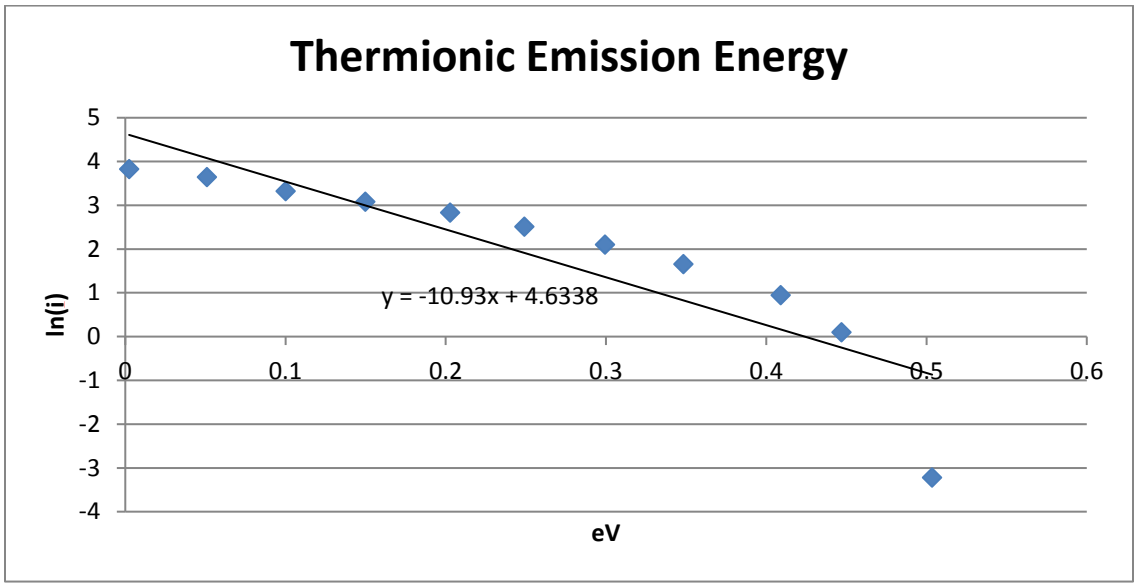


Figure 5: Plot of the natural log of current against the retarding potential V

E. Analysis and Discussion

Upon inclusion of the previously discussed systematic error, the voltage for the cross-section minimum is now 1.175, which is far closer to the expected value. This also means that my calculated radius is closer, it now being 2.86^{-10} cm. Still, the corresponding electron energy value for 1^{-10} cm is 10eV for the 1D interpretation. So, I tried using the final values with the 3 dimensional spherical square well solution I previously discussed. Plotting equation 4, I see the following graph:

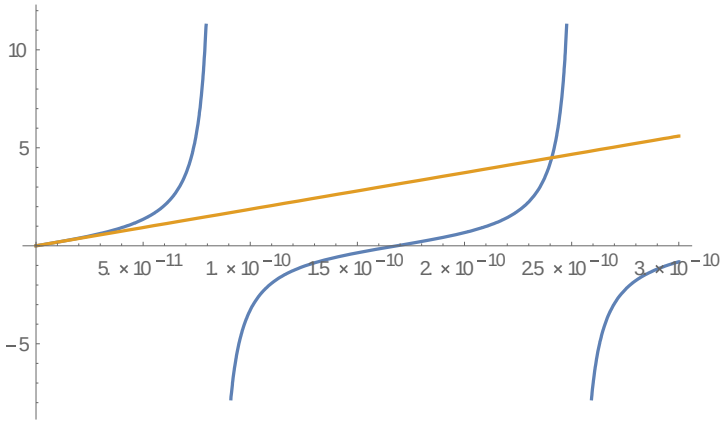


Figure 6: Plot of $\tan(k'a)$ and $k'a$ where a is represented on the x-axis in order to find the first intercept that is non-zero. The k' value that I calculated uses the binding energy of the outermost Xenon electron shell, which is 12.1 eV.

The first intercept is around 2.4^{-10} m, which is close to the actual radius. My assumption that the potential well is the outermost electron energy level isn't too off; however the energy in a noble gas atom isn't uniformly distributed. A coulomb screened potential would be needed, and this is mentioned by Kukolich[1] in his article. It would account for the different energy levels of the atom, which would lead to a more accurate final result. Unfortunately, due to time constraints I wasn't able to solve for scattering off of a 3 dimensional screened coulomb potential.

Though my final value after including systematic error is within 17.5% of the expected value, I felt that there a few issues with the experiment. There was room for plenty of error with the frozen Xenon. As my setup had the liquid nitrogen evaporate rather quickly, it probably affected the amount of scattering that actually occurred. On average, the liquid nitrogen was replaced around every fifteen minutes. During that time, the Thyratron had time to heat up, as the heater inside the tube produces significant thermal energy. When the tube was returned to the liquid nitrogen, I did wait for the Xenon to freeze out, as generally the current would increase as the scattering decreased to zero. However, this repeated process might have somewhat skewed the data, as we can see in Figure 2 that the plot isn't entirely linear.

When testing to find the contact and thermionic potentials for the Thyratron, the small current would vary wildly, making measurements difficult. I on multiple occasions would wait for the current to stabilize, but then notice it began changing again as the liquid nitrogen had evaporated and the Thyratron was heating up again. This lead to possible errors in measurement which might of affected the final values. Though this is a small issue as my final voltage was decently accurate, it's something that should be considered if the experiment is done again.

Also, previous tests had the heater operating at four volts. As stated, the lower voltage is to mitigate space charge effects, resulting in less noise. At five volts it is possible that the space charge effects increased enough to introduce more noise. This effect limits the amount of electrons that can be thermionically emitted, which results in changes in the electron beam. Another test could easily be done to see if this is indeed the case.

II. Electron Diffraction

A. Background Information

1. Slit Experiments

Any physicist will know about the single and double slit experiment, and has a general understanding about how both work. So I will only briefly go over these topics. Single slit diffraction is the result of the interference caused by the wave as it passes through the slit. As such, we can see distinct bright and dark spots from the propagated wave. In the two slit experiment, the interference creates shaper bright bands in the resulting wave. Since electrons possess wave like properties, they will react similarly with slit diffraction. Looking at the intensity curve though, we see that for the double slit experiment, the individual intensity peaks is superimposed under the curve for single slit diffraction.

This continues on with multi-slit diffraction. By increasing the number of slits the intensity peaks will become sharper and sharper. The diffraction intensity will still be a convolution of the sharp

individual multi slit peaks and the single slit curve. Univeristy of Cambridge[5] has this great image showing the convolution of the peaks.

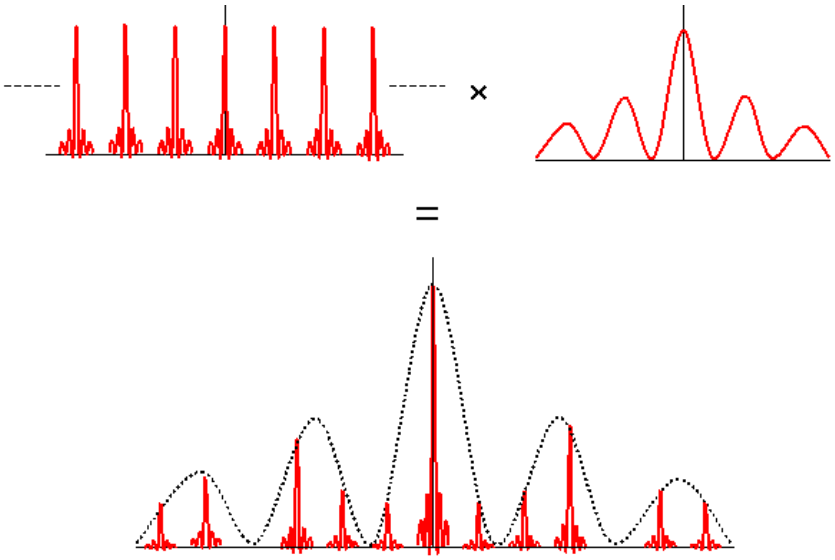


Figure 7: Convolution of a multi-slit diffraction pattern with infinitesimally small slits on the top-left, and a slit of some width on the top-right. The bottom one is the resulting diffraction pattern

With multi slit diffraction, looking at the distance between the central bright spot and first peak, we can find an equation relating the wavelength of the wave to the slit separation[6].

$$dsin(\theta) = \lambda$$

(Equation 5)

Here, d is the slit separation, θ is the angle of separation, and λ is the wavelength of the electron. There is also another relationship relating the length of the diffraction tube and the distance between the peaks and the angle of separation[6].

$$\tan(\theta) = \frac{x}{L}$$

(Equation 6)

With this relationship where x is the distance of the first maxima and L the distance from the slit to the projected image, I can easily find the angle of separation. Plugging that angle into Equation 5, I can then find the slit separation.

2. Diffraction of a Lattice Structure

Crystals act in a similar manner to a diffraction grating. The distance between the atoms can be seen to serve as the slit separation, and it's more specifically the distance between planes in the crystal. Due to the number of atoms in the crystal, it can be said to possess n-slits, giving rise to sharp peaks in the resulting diffraction pattern. However, diffracting off a crystal lattice is slightly more complicated than a 1 dimensional diffraction grating, as the 3 dimensional crystal gives rise to a complex diffraction

pattern. Instead of seeing a single straight line for the pattern, diffraction off the crystal results in a 2 dimensional image where the pattern essentially radiates from the central beam. Also, in this situation the sample used in diffraction is polycrystalline, which means that there are many orientations of the lattice structure; the rings will be at varying distances due to the different planar spacing. The analysis however, is still the same, and I can use the relationship between the peaks shown in Equation 5.

Now, since I am diffracting off Aluminum, I have to understand its crystal structure. University of Colorado[7] has a good diffraction experiment where they talk about the lattice structure of aluminum, which I used for my purposes. As it turns out, Aluminum is a face-centered cubic[7], which means that the crystal is a perfect cube that has an extra Aluminum atom in the center of each face of the cube. This however does make figuring out which plane the first ring will represent slightly difficult.

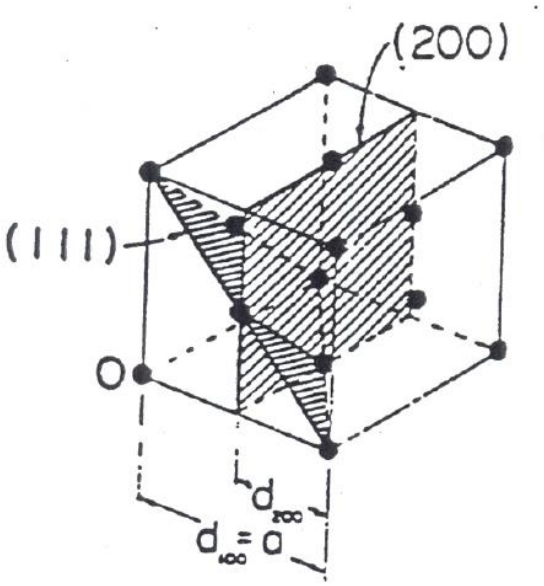


Figure 8: FCC lattice structure of Aluminum.

In the University of Colorado[7] lab handout, they talk about miller indexes, which serve as a ratio that denote separate planes in the crystal. Figure 8, taken from their handout[7], shows the crystal structure of Aluminum, but also two miller indexes. These are two of the multiple planes that exist in the crystal, both of which result in diffraction. Now, I need to figure out which of these planes results in the first ring. As it turns out, there is a relationship between the intensity of the diffraction peaks and structure factor, something related to the miller indexes. This results in only certain planes being diffracted, and as it turns out, the 111 plane is what corresponds to the first order ring. I found this out after I measured the distance to the first ring, and used that plane to then calculate the actual length of a side of the cube. Though the planes are 3 dimensional, looking at it from a certain angle presents a 2D representation.

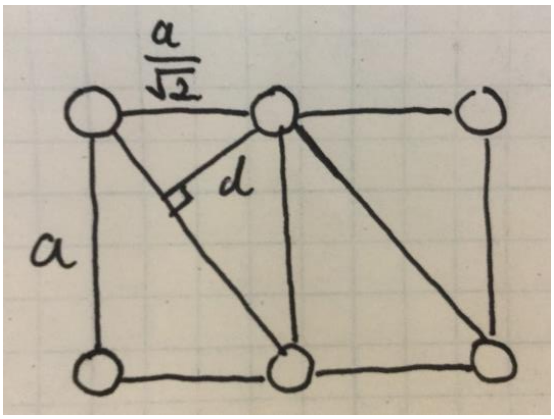


Figure 9: 2D representation of the lattice structure where the distance between the planes is apparent.

In this image, d is the distance between planes, and a is the length of the cube. With some geometry, an equation relating d to a can be found.

$$a = \frac{\sqrt{2}d}{0.816}$$

This constant a is the lattice constant, and it is a known value. For aluminum, the lattice constant is 4.046 Å[8].

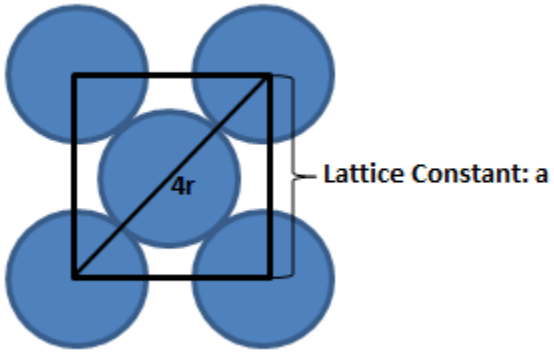


Figure 10: Representation of the lattice constant and the radius of the bonded aluminum atoms in the crystal.

Looking at Figure 9, we can see a distinct relationship between the radius of the atoms and the lattice constant. With a little geometry, we can see that the radius of the atom is equal to:

$$r = \frac{a}{2\sqrt{2}}$$

So, this shows that if I find the lattice constant, I can find the radius of the atom.

3. Convolution of Diffraction Patterns

Another way of finding the radius of the atom would have been to try and graph the intensity of the aluminum diffraction pattern. Since this is still a diffraction pattern, the resulting intensity peaks still fall under the Bessel curve of a single slit diffraction pattern, like Figure 9. If could take a photo of the diffraction pattern, I could than try and analyze that photo to get an intensity curve for Aluminum. Using this curve, I could then pretty easily work backwards to find single slit intensity envelope, which is related to the radius of the atom. Unfortunately, I wasn't able to get a photo which allowed me to get distinct intensity curves, and as such I couldn't use this method to find the radius of the atom.

B. Aluminum Diffraction Patterns

1. Camera Setup

Figure 11: Photo of my camera and the electron diffraction tube

For this experiment, I used my Canon EOS Rebel SL1 to take photos of electron diffraction patterns. A Sargent-Welch electron diffraction tube was used, and all the photos were taken in the dark. I took many pictures over the course of the final week, resulting in a few hundred photos of the aluminum diffraction pattern. Generally, I would try and vary the voltage, intensity and position of the beam to find

a diffraction pattern with sharp peaks and good resolution. I would then take photos at lower intensities with a long exposure time in order to acquire distinct bright spots. Brighter rings in comparison with the background result in better intensity analysis in the image processing software I used, ImageJ.

When I wanted to measure the distance between rings, I removed the thick glass protective cover of the diffraction tube, and placed a transparent ruler against the pattern. Then, a photo was taken, and using ImageJ I could accurately find the radius of the rings.

2. Diffraction Patterns

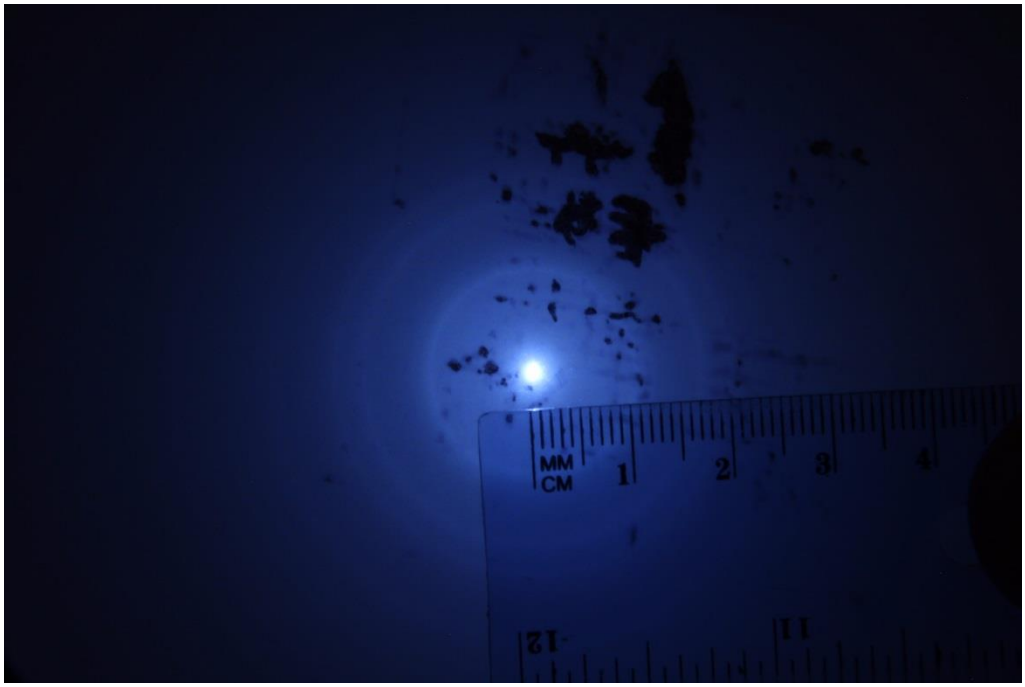


Figure 12: Photo of Aluminum diffraction pattern. This was taken at 7900 eV.

For an electron energy of 7900 eV, the radius of the first ring was found to be 1.028 cm. Using equation 6 and a tube length of 17.34 cm, I calculated the angle of separation. Since I knew the electron energy, I found the wavelength and could then use Equation 5 to derive the separation between the crystal planes. That distance is 2.33 Å. I then worked from that to find the lattice constant as previously discussed to find the lattice constant, which in my experiment is 4.038 Å. This is very close to the actual value of 4.046 Å.

I then repeated this calculation for various other planes to make sure that I was looking at the right plane for diffraction, and the 111 plane was the only one to provide a similar lattice constant. I know this to be the case since all my measurements are accurate, I can assume that my calculation should give a reasonable value for the lattice constant. Since I had the right plane of reflection, I can then calculate the radius of the atom, which I found to be 1.43 pm. This is the same value for the metallic radius of aluminum when it's a crystal.

C. Analysis and Discussion

It appears that electron diffraction is a pretty accurate way of measuring the radius of the sample which it is scattering off of. The calculations ultimately are simple and straightforward, and resulted in a very accurate measurement of the radius. However, I did want to use the method of convolution to find the radius. Due to time constraints though, I was not able to find an image of sufficient quality and brightness to get that intensity curve. A main issue is that the tube is old, so getting a great pattern is difficult, but possible. More time would be needed to see if I could possibly get a better image of the pattern. Otherwise, electron diffraction does work as a sufficient way of acquiring the radius of aluminum atoms in a crystal lattice.

III. Conclusion

Overall, everything went rather well. Both experiments can be used to find atomic radius, and both provided values that were very reasonable. The Xenon Thyratron experiment lead to a final value of around 2.4^{-10} m, which is close to the actual value of 1.08^{-10} m. With more time, I could find a more accurate result with a coulomb screened potential, but the overall experiment is a success. It demonstrated the desired quantum mechanical effects that lead to me being able to derive the Xenon radius. If the experiment is repeated though, I would recommend that a power box is built for the experiment to allow for more accurate and easier change in voltage. The variable power supply I used wasn't extremely sensitive, and it took time to get the 0.05 volt increments I wanted. Also, a dewar flask should be used instead of my Styrofoam cup, to allow for better measurements.

For electron diffraction, the experiment was accurate. I found a value for the radius that corresponds exactly to the radius of the Aluminum, exemplifying the efficacy of electron diffraction for such purposes. The next step would be to get an intensity curve off the diffraction pattern. This would just simply take more time with the diffraction tube that Cornell has. Better yet, a tube without as much use would give a much brighter and more distinct pattern, which would lead to better image analysis.

IV. Works Cited

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