Steering audio beams using bucket brigade delayed phase shifting

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This paper outlines work done to steer audible sound frequencies using a two dimensional phased array. Nonlinear acoustic theory describes the means to create audible sound from ultrasonic frequencies through heterodyning. Given enough power, amplitude modulated (AM) ultrasound will produce a loudspeaker from the air with extremely narrow directivity. This modulated ultrasound can then be steered by passing it to elements of a phased array. Digital steering is unfortunately limited by its prohibitively excessive required bitrate and degrees of parallelism. An analogue delay schema is proposed utilizing tapped bucket brigade topology.

I. INTRODUCTION

The directivity of a loudspeaker is gated by the frequencies of the waves it is required to emit. Tones audible to the human ear have relatively long wavelengths, and as directivity is proportional to the ratio of the size of the emitter to the length of the wave, a practical loudspeaker will emit sound essentially omnidirectionally. One novel solution to this problem is through the utilization of ultrasonic speakers, emitting frequencies inaudible to the human ear. Audible frequencies can piggyback on the ultrasound passed to the speakers, attached in much the same fashion as an AM radio transmitter. Due to the nature of nonlinear acoustics, audible sound is emitted from the ultrasound itself, allowing for highly directed beams of audio.

Achieving a sharply directed beam begs a further question: how can the direction of the beam be changed? If a listener using an ultrasound-to-sound speaker moves from the audio beam, the speaker must be manually rotated to allow for uninterrupted listening. This problem may be solved through the use of an ultrasound-to-sound phased array, which allows for electronic steering of the audio beam. Phased arrays consist of elements that emit with a calculated phase difference between each other, resulting in a directed beam along the phase-aligned angle.

This solution carries with it further complications, most stemming from the nature of the waveforms being steered; sound is generally composed of a large spectrum of frequencies, each with different phasing requirements. This is generally combated with the use of true time delay (TTD) phase shifters, which make use of the frequency independent correlation between time and phase.

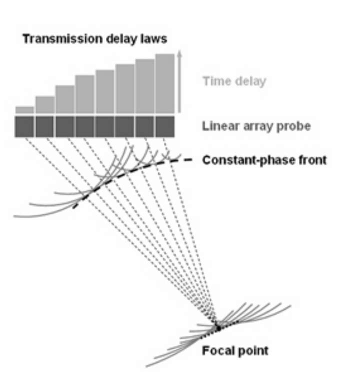


FIG. 1. Simplified implementation of a TTD phased array

TTDs are simple in theory, but can be difficult and expensive to implement in practice; each array element requires a fed sample rate greater than the Nyquist frequency ($2f_{max}$), and because each element operates on a separate delay line, the minimum required throughput for the system is equal to

$$2NRf_{max} \tag{1}$$

where N is the number of elements and R is the resolution of each sample in bits. For 25KHz ultrasound with a bandwidth of 5KHz, a resolution of 8 bits/sample, and just 16 elements, this requires over 15 Mbps of throughput. For higher quality audio, 40KHz ultrasound, and many more elements, implementation becomes extremely costly.

This paper poses a tapped analog delay scheme as a solution for phased ultrasound-to-sound speakers. The delay lines constructed, known as Bucket Brigade Devices (BBDs), were first engineered in 1969 and were used predominantly for guitar echo effect pedals. They were rapidly replaced by digital delay devices, as BBDs were limited by many compounding factors. BBDs are composed of hundreds, or thousands, of delay stages each comprised of a capacitor and two transistors. These capacitors would leak some amount of charge backwards after every stage, corrupting the signal and limiting the number of stages. The length of the delay was limited further by the delay rate being the same as the sample rate, meaning that low frequencies (and therefore, longer delays) had very small practical bandwidths. These factors, however, are not of particular concern in the construction of linear phased arrays, as the number of needed delay stages is relatively small, and only very short time delays are desired.

II. THEORY

Ultrasound propagating through a nonlinear medium will exhibit nonlinearities itself; these nonlinearities can be expressed as a Taylor series. If the nonlinearity is weak, as it is in air, only the first nonlinear term is of concern; this term is proportional to the square of the waveform. Assuming our waveform consists of two frequencies, this yields

$$\Psi_{nl} \propto (\sin(\omega_1 t) + \sin(\omega_2 t))^2 \tag{2}$$

This expands to contain the following components:

$$\cos((\omega_1 + \omega_2)t) + \cos((\omega_1 - \omega_2)t) \tag{3}$$

Clearly, this nonlinearity will produce two other tones at frequencies equal to the difference and sum of the input frequencies. This effect is the cornerstone of the ultrasound speaker.

The means of beam steering via phasing may be derived by examining the wave pattern created by an N element linear array. The amplitude at any given point in the far field can be expressed as:

$$p = Ae^{i\omega t} * \sum_{n=1}^{N-1} e^{in(kd\cos\theta + \beta)} \tag{4}$$

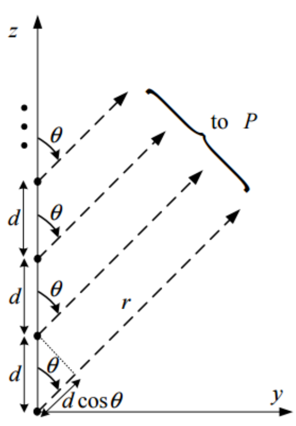


FIG. 2. Illustration of N element array

Here, β is the progressive phase delay between each element, and d is the inter-element spacing. To determine the effect of the array on the farfield pressure, only the summed term need be considered. For simplicity, this term will be called the *array factor*, or AF, and $\psi = kd\cos\theta + \beta$. Several steps are taken to remove the sum:

$$AF * e^{i\psi} - AF = e^{iN\psi} - 1$$
$$AF = \frac{e^{iN\psi} - 1}{e^{i\psi} - 1} = \frac{e^{iN\frac{\psi}{2}}(e^{iN\frac{\psi}{2}} - e^{-iN\frac{\psi}{2}}}{e^{i\frac{\psi}{2}}(e^{i\frac{\psi}{2}} - e^{-i\frac{\psi}{2}})}$$
$$AF = e^{i\frac{\psi}{2}(N-1)}\frac{\sin(N\frac{\psi}{2})}{\sin(\frac{\psi}{2})} \tag{5}$$

The term on the left is a phasing factor that is only meaningful when considered alongside emitters outside the array. This leaves the final array factor, normalized as

$$AF = \frac{\sin(N\frac{\psi}{2})}{N\sin(\frac{\psi}{2})} \tag{6}$$

This function contains both major and minor lobes, describing the directivity of the array. For meaningful steering, it is at least required that only one major lobe exist in the positive plane ($\theta = 0$ to π), regardless of steering angle.
This requires an interelement spacing $d < \gamma/2$.

The above function also yields the required time delay. Assuming that the number of major lobes is one, the only maxima will occur when $\psi = kd\cos\theta + \beta = 0$. From this, it is apparent that the steering angle may be controlled by the progressive phase difference, by setting

$$\beta = -kdcos\theta \tag{7}$$

An equivalence between phase and time may be derived from this equation.

$$\beta = -kd\cos\theta = -2\pi\frac{d}{\gamma}\cos\theta$$
$$\beta = -2\pi\frac{df}{v_s}\cos\theta$$
$$\frac{-\beta}{2\pi f} = \frac{d}{v_s}\cos\theta \tag{8}$$

The right half of the final result above is a constant, in units of time, showing the desired phase/time independence. Time delays can be easily calculated by simply passing the desired steering angle into the cosine argument.[1]

III. METHODS

To achieve audio with low levels of distortion, several stages are required.

A. Mixing

First, the audio must be passed through a modulator, as the final audible waveform will be heavily demodulated as dictated by the KZK equation:

$$\frac{\partial^2 p}{\partial z \partial \tau} = \frac{c_0}{2}\Delta_r^2 p + \frac{\delta}{2c_0^3}\frac{\partial^3 p}{\partial \tau^3} + \frac{\beta}{2\rho_0 c_0^3}\frac{\partial^2 p^2}{\partial \tau^2} \quad (9)$$

This differential equation models nonlinear effects in sound beams, and is, in general, impossible to solve. However, the KZK equation can be solved conditionally. The equation for the linear part of the ultrasonic field is first solved, using the following as a pressure model

$$P_0 E(t)\sin(\omega_0 t) \quad (10)$$

where P_0 is the ultrasound amplitude and $E(t)$ is the envelope containing the modulated sound pattern. This equation, alongside a far-field assumption, yields the equation

$$p_{nl} = \frac{\beta P_0^2 a^2}{16\rho_0 \alpha c_0^4 x}\frac{d^2}{d\tau^2}E^2(\tau) \quad (11)$$

where β is the nonlinear coefficient, a is the transducer area, and α is the absorption coefficient of the ultrasound in air. This approximately describes the nonlinear pressure distribution in the farfield.[2] Of particular note is the P_0^2 term in the numerator. This shows that the magnitude of the nonlinear effect is proportional to the square of the ultrasound pressure. Basically, the greater the intensity of the ultrasound, the greater the volume of the sound.

The main purpose of this derivation was to find the dependence of the nonlinearity on the audible sound. This is given by $\frac{d^2}{d\tau^2}E^2(\tau)$. The sound is squared and differentiated twice before demodulation. Thus, to achieve the lowest possible distortion, the audio must be square rooted and integrated twice before being mixed into the ultrasound.

The second stage combines audio and ultrasonic frequencies to achieve the desired heterodyning. For this, the audio is amplitude modulated, using 25KHz ultrasound as a carrier frequency. Dual band AM can be produced simply through multiplying the desired signals and adding the carrier. Taking the Fourier series of this resultant waveform yields a large spike at the carrier frequency and bands immediately above and below, at the difference between the signals. These waves will naturally demodulate in the air to produce our audible sound.[3]

B. Delay

The third stage is the BBD delay circuit. To construct a simple bucket brigade stage, only two parts are required: a capacitor and a MOS transistor. The gate of the transistor is connected to an external clock, the source is connected to the input, and the gate is connected to the capacitor. When the clock switches to its positive state, the transistor activates and lets an input voltage through to the capacitor. To create additional stages, a clock inversely synchronized with the first is required. When the second clock ticks upward, the first ticks downward; this lets current flow across the second transistor to the second capacitor, while preventing backwards current to the previous stage.[4]

This was the first iteration of a BBD circuit, which was later improved through the addition of a second MOS transistor, gate DC biased, in series with the first. This vastly reduces feedback into the previous stage, allowing for more stages without significant distortion.[5] For the purposes of this experiment, only a few stages were needed.

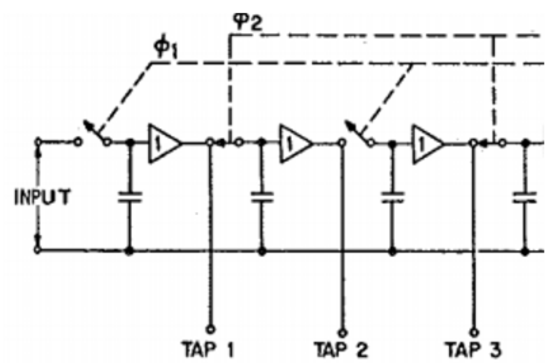


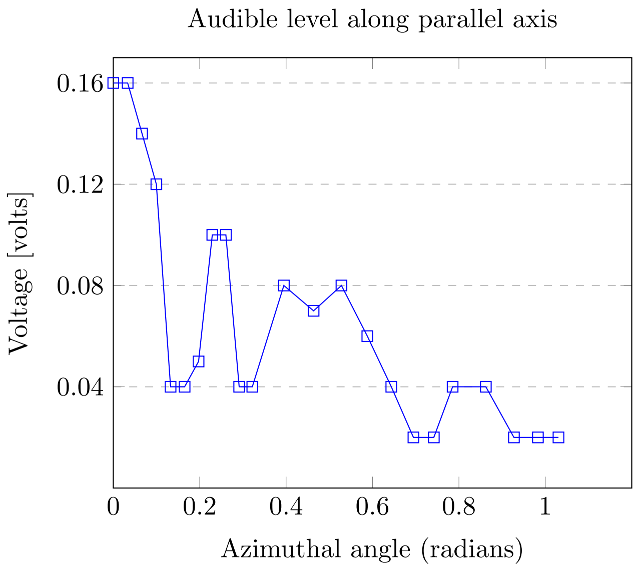
FIG. 3. Equivalent circuit for simple bucket brigade device

IV. EXPERIMENT

The mixer was built first with a simple cascaded double integrator. For this experiment, it was chosen not to square root the audible envelope before modulation, as the available transducers had prohibitively poor frequency response to capture the larger bandwidth of a square rooted AM waveform.

The amplitude modulator was built from an AD633 multiplier, with the carrier signal fed through a potentiometer to the summing input to allow for a variable modulation index. This is required to adjust for input signals of varying amplitudes, and to reduce the harmonic distortion caused by the squared resultant signal.

This simple circuit was tested through a parametric array built from 85 25KHz transducers. The resulting nonlinear effect was very audible, and had noticeable directivity in the near field.



This data agrees acceptably closely with theory, which approximates the directivity with a first order Bessel function.[6]

Next, the modulated signal is passed through a second order active band-pass filter to reduce noise, unwanted subultrasonic frequencies, and aliasing frequencies above the Nyquist limit of the delay sampling clock.

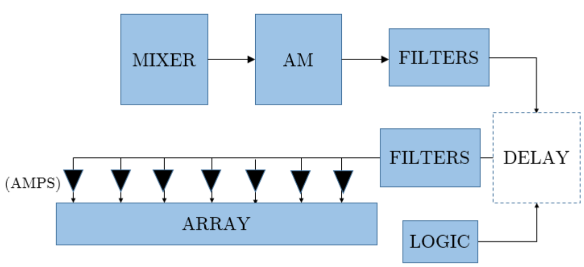


FIG. 4. Block diagram for complete array

The delay line was built with a series of sample and hold ICs, with alternating elements' logic inputs fed by a variable frequency two phase clock made from NOR gates. Several limitations arose from this implementation.

Firstly, the acquisition time of the holding capacitor can only be driven so low without introducing unwanted distortion. This limited the delay time to a 2.5 μs per element, corresponding to a minimum steering angle of approximately +/- 10 degrees off center.

Secondly, the delay clock and the sampling clock for the BBD are the same, resulting in a maximum single step delay. The highest frequency desired to be present in the AM wave was 30 KHz, demanding an absolute minimum 60 KHz sampling rate. The single step delay is equal to $\frac{1}{2f_s}$, or 8.33 μs. This corresponds to a maximum steering angle of approximately +/- 37 degrees off center. This limited available steering can be remedied through the use of additional sample and hold elements per delay tap, a solution which was not pursued due to time and spacing constraints.

In the time allotted to the author for the experiment, the filters, delay line, and amplifiers for a 12 element BBD were built. Testing, however, was limited and unsuccessful. This was most likely due to the fairly hefty complexity of the system, and would be remedied given more time. To decrease system complexity, an analog BBD IC could be implemented, replacing the S/H delay line entirely. For the purposes of this experiment, the Reticon TAD32A is ideal. This part is exceedingly rare, however, and can be difficult to obtain.

To prevent issues with wire crossing, reduce parasitic capacitance and output resistance, and to save time in the long run, several custom PCBs were modeled in Eagle CAD, converted to G-Code with pcb-gcode, and passed to a CNC mill. Modules encompassing several stages of the design were mapped onto double sided boards, and compiled onto three G-Code files corresponding to copper traces for both sides and drill holes. Initial runs for the modulator module were successful, but further milling was suspended due to time constraints.

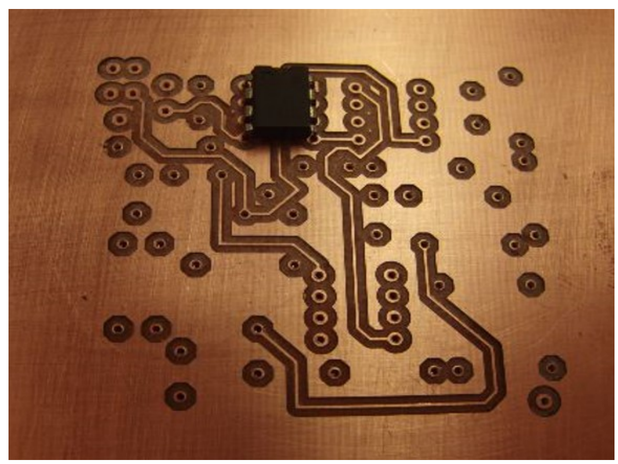


FIG. 5. One side of the modulator module, milled onto a copper clad

V. DISCUSSION

An analog approach to the implementation of audio frequency phased arrays in air was proposed. Time constraints allowed only limited testing of the principles of nonlinear acoustics, which was demonstrated through an 85 element parametric array. This testing was successful, and demonstrated noticeable directivity along the azimuthal.

Future potential for work is extremely high, as the state of this experiment remains unfinished. For immediate further research and experimentation, the delay and amplifier modules will be milled and tested, and combined with the modulator module to form a complete acoustic phased array. This device could be further optimized with more specialized transducers and further modulation, and system bulkiness and complexity reduced through the use of BBD ICs and single supply schemes. If completed and tested successfully, the acoustic phased array could not only serve as a novel demonstration of acoustic principles, but also have many potential commercial uses as a source of personalized, aim-able audio in low noise environments.

VI. REFERENCES

[1] Nikolova NK. Lecture 13: Phased Array Theory - Part I; 2016. Available from: http://www.ece.mcmaster.ca/faculty/nikolova/antenna_dload/current_lectures/L13_Arrays1.pdf.

[2] Pompei JF. Sound from ultrasound: the parametric array as an audible sound source. MIT; 2002.

[3] Yoneyama M, ichiroh Fujimot J, Kawamo Y, Sasabe S. The audio spotlight: An application of nonlinear interaction of Sound waves to a new type of loudspeaker design. J Acoust Soc Am. 1983;73(5):1532–1536.

[4] Weckler GP, Buss RR; Reticon Corporation. Bucket Brigade Devices Circa 1977. 1977;Available from: http://www.imagesensors.org/Past%20Workshops/Marvin%20White%20Collection/1977%20Short%20Course/1977%203%20Weckler.pdf.

[5] Sangster FLJ. Integrated bucket-brigade delay line using MOS tetrodes. Philips Technical Review;31:266.

[6] Tucholski E. Directivity Index and Multi-element Arrays;. Available from: https://www.usna.edu/Users/physics/ejtuchol/documents/SP411/Chapter14.pdf.