Structural Resonance and Damping

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Introduction

Earthquakes are very powerful and difficult to predict. This is why they cause some of the deadliest and costliest disasters in the world. Although it is difficult to predict when an earthquake will hit and how big it will be, it is easy to see where they are most likely to occur. Areas like the west coast have very strict building codes to help prevent building collapse because those areas have a very high likelihood of experiencing an earthquake. Our research will look into areas that have a risk for earthquakes but not as frequently as other areas. These places have buildings that were not build with as strict building codes leaving the people who occupy those buildings at risk. We will model a building in one of these places, analyze its modes, and explore means of making it safer.

Theoretical Background

When looking at a damped harmonic oscillator that is subject to an external driving force, it can be represented by the equation of motion

$$m\ddot{x} = -kx - c\dot{x} + F\cos wt$$

where the force that is exerted on the system is represented by *Fcos wt*, the mass is *m,* the spring force is *k,* and the damping force is *c* [2]. A phenomenon called resonance occurs when the driving frequency is very close or matches the natural frequency, $w_0 = \sqrt{k/m}$ of the system [1]. For theoretical purposes, this system is represented without damping by the equation

$$m\ddot{x} = -kx + F\cos wt$$

Now, if $w = w_0$ in the driven, undamped oscillator, we can see that there will be a catastrophically large response. The steady-state amplitude at a resonant frequency with damping can be seen in the equation

$$A_{max} = \frac{F/m}{2\gamma\sqrt{w0^2 + \gamma^2}}$$

where $\gamma = c/2m$ [2]. In the case of week damping this equation can be simplified to

$$A_{max} = \frac{F/m}{2\gamma m w 0}$$

showing that the amplitude of the oscillations at the resonant condition become extremely large if the damping factor γ is small and that the amplitude is small if the damping factor is very large [2]. In mechanical systems, it is very important to recognize this and have the appropriate damping factor to limit large resonant amplitudes. In other types of systems however, it can be beneficial to obtain these large resonant amplitudes like attenuating a signal.

There is a quality factor that represents the sharpness of a resonance and it is Q. A high Q-value results in a sharp resonance. Q can also be thought of as the natural frequency over the full width at half of the maximum on the amplitude versus frequency graph [2].

$$Q = \frac{w}{2\gamma} \text{ or } Q = \frac{w0}{\Delta w}$$

A large Q-value can be extremely useful in electrical systems but is to be avoided in mechanical systems. If a mechanical system has a high Q value, it is at risk of hitting a resonant frequency and experiencing catastrophic results. By obtaining a low Q value the mechanical system is at risk to experience more resonant frequencies, but these will not be as sharp, resulting in smaller amplitudes of oscillation [2].

Even after engineers have implemented proper damping techniques to earthquake-prone buildings, they still receive large amounts of vibrational energy. The material that a building is made up of plays a big role in damping the horizontal vibrations caused by earthquakes [3]. Building made solely of brick and concrete are at a greater risk of crumbling during an earthquake than buildings made of steel and reinforced concrete. Materials like steel can absorb a lot more of the vibrational energy, allowing the building to bend a considerable amount without breaking [3].

Engineers have come up with a few ways to damp buildings. The two main damping techniques are structural bracing to stiffen the walls and base isolation in the foundation of a building [3]. Cross bracing is a technique that has been used for many years. It involves placing two diagonal beam in an X-shape on a wall. Horizontal components of buildings such as floors and a roof are often strengthened with trusses which are made of diagonal structure members. Another way engineers stiffen the walls of a building is by placing shear walls in parts of the building that have few openings like elevator shafts or stairwells [3]. However limited flexibility in walls can become a serious problem. Moment-resisting frames are often used which allows the beams and columns to bend but the joints are designed to be rigid [3].

A buildings foundation is perhaps the most important section. It is crucial for buildings, especially ones in earthquake zones, to be built on solid, firm soil. If this is the case a damping

technique called base isolation can be used. This is essentially floating a building above its foundation on a system of bearings, springs or padded cylinders. The bearings would be attached to the building and foundation by steel plates. This allows the ground and therefore the foundation to be displaced during an earthquake without moving the structure above it greatly reducing the horizontal acceleration of the building [3].

A technique that is still being tested is installing active mass damping systems on certain floors of buildings. These systems consist of a heavy mass mounted on a floor connected to dampers that act like shock absorbers. The mass is can move independently of the building. This means that when the building begins to oscillate, the mass will move in the opposite direction [3]. This will help reduce the amplitude of the oscillations of the building.

Experimental Procedure

The first step was to decide on a building to model and perform the experiment on. We thought about a few things when making this decision: location, size, and information. It was important that the building was in an area where an earthquake could occur. We also wanted to model a building that was not necessarily earthquake-ready. We wanted to the building to be large enough to be able to easily see the modes but not too big to where we would be unable to scale the building down to a size that we could work with. Lastly, we needed basic information on the building such as: when it was build, floor plans, material of its structure, and basic dimensions. The building that we chose was the 100 North Main Building in the Memphis, Tennessee.

Figure 1: 100 North Main Building, located in Memphis, TN [5].

Figure 2: Scaled model of the building made of balsawood.

To model the building we had to decide on an appropriate scale of the actual building. When modeling something as complicated as a large building, it is necessary to simplify the model and make certain assumptions. We chose to model the skeletal structure of the building which was made entirely out of steel. Our model assumes that every beam used was a 1 foot x 1 foot I beam and that the building was just over 400 feet tall (actual size 427 feet). We used $\frac{1}{8}$ inch x $\frac{1}{8}$ inch x 48 inch balsa wood as a material to build the model. This meant our

model would best fit a 1/100 scale of the building. Balsa wood was chosen for a few reasons: it fit the scaled dimensions of the building, it was easily accessible, and the relationship of its properties made it simple to scale the natural frequency of it to that of the actual building.

$$\frac{1}{l}\sqrt{\frac{Y}{\rho}} \propto f$$

From this relationship we estimated that the balsa wood model would have roughly ninety times the natural frequency as the simplified full scale building.

The model was able to be created using the CAD software, PTC Creo Parametric 3.0 before we constructed it out of balsa wood. The model created using PTC Creo Parametric 3.0 was the full scale simplified version. The benefits of using computer software to first model and simulate model are that is very easy quick to make adjustments that one would not be able to do or notice when physical building a model and the model can be simulated using the computer software. This model allowed us to estimate material needed as well as other physical characteristics that would otherwise be difficult to calculate by hand. We were also able to come up with a visual display of the modal analysis of the structure.
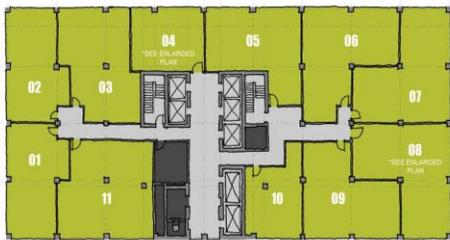

Figure 3 (top): The floor plan of the upper floors of the 100 North Main Building [6]. Figure 4 (bottom): A floor being assembled on a drawn out grid.

The scale model was constructed out of 36 inch and 48 inch pieces of balsawood. The first step in the building process was estimating and collecting the materials we need to complete the building. Once we had a good idea of how many pieces of each length of wood we need, we began to cut all of the pieces. As we had enough cut pieces, we assembled the individual floors on a sheet of paper that had the floor grid drawn out on it. Each joint on the model was fastened using hot glue.

Given the number of joints and time to complete the project, it was decided that hot glue would be the best option because it quickly formed a solid bond even though its physical properties would be much different than the beams. After all floors were assembled, vertical beams were placed in each corner of the first two floors' grids to replicate the actual building's structure. Each vertical beam was marked to indicate the interval of each floor so we knew how far apart each floor would be separated.
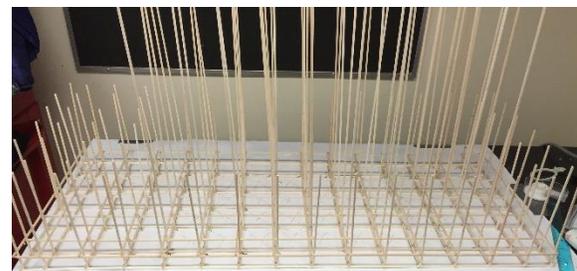

Figure 5: Vertical beams assembled into the corners of the grids of the first two floors.

Once all vertical beams had been positioned, each additional floor was placed one by one and glued in place.

In order to vibrate the building in an attempt to simulate an earthquake, we needed to build a shake table. We decided that the best way for us to drive the system was with a Pasco mechanical vibrator controlled by a Pasco signal generator. It was also necessary for the entire building to be able to fit on the shake table so we decided to use a 1 ¼ inch thick piece of insulation to anchor the building to. As a base, we used a ½ thick sheet of particle board. The key to the shake table to work accurately was to have as little friction as possible between the two sheets. We evenly placed eight magic sliders on the sheet of insulation and eight 6 inch x 4 inch felt pads on the sheet of particle board. The mechanical vibrator was attached to the sheet of insulation by an arm that was slid in a slit cut into the sheet of insulation. Rubber bands were also attached at the corners of the insulation to help the shake table move in one direction.

Upon completion of construction of the model and shake table, we vibrated the structure through a broad range of frequencies looking for spikes in the acceleration of different sections of the building. Acceleration was measured, using 18 accelerometers strategically placed throughout the model. When the accelerometer chips were moved, they put out an analog voltage that was converted to digital through a NIDaq board. A program was created in LabView 2014 SP1 to interpret and plot the converted voltages that the accelerometer chips had put out.

Results and Discussion

Using PTC Creo Simulate we were able to perform finite element analysis on the CAD model of the full scale skeletal structure. From this analysis, we were able to visualize the modes of the structure. As expected, the top of the building was found to have the most displacement in the first few modes. We are looking more in depth at the first three modes (shown below) because we expected to be able to excite those modes in the physical model that we built. Since we are only looking at the first few lower modes, our model can be simplified to a solid bar that has one end fixed. The bar will act very similar to the building at these modes. This can make visualization and calculations much simpler than the simplified yet very complex model that we looked into using the PTC Creo software.
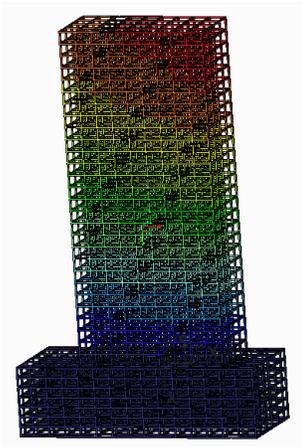
Figure 6: Mode 1 of the original CAD model.
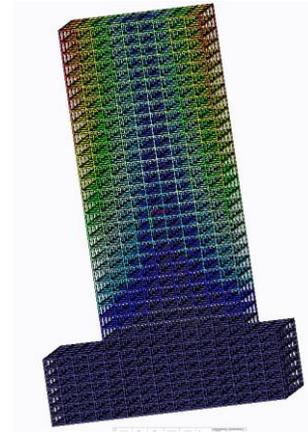

Figure 7: Mode 2 of the original CAD model.

Figure 8: Mode 3 of the original CAD model.

The first mode was calculated to be excited at 3.82 Hertz and sways at an unevenly across the floors. This is due to the top part of the building not sitting directly on the center of the base. The second mode should be excited at 4.41 Hertz and it sways evenly throughout the floors. The third mode can be excited at 4.53 Hertz and is different from the other two because it shows a twisting motion of the building. From the relationship we came up with to scale the frequencies, our model should excite these modes at 343 Hertz, 397 Hertz, and 407 Hertz.

Using the shake table, we were not able to see these modes at the projected frequencies. However, we were able to find several resonances at much lower frequencies. Around 6 Hertz, we were able to physically see resonance throughout the building. The top of the building was moving significantly more than the base as well as twisting which gave us a major clue that this was the third mode. After running a Fourier transformation of an impulse applied at the base of the structure, we were able to confirm that there was a mode around 6 Hertz. We were also able to find some higher frequency resonances with the Fourier transformation. Looking more closely at a single accelerometer located at the top of the building, we analyzed how the sharpness of the resonance or Q-value would be affected by different types of mass damping.
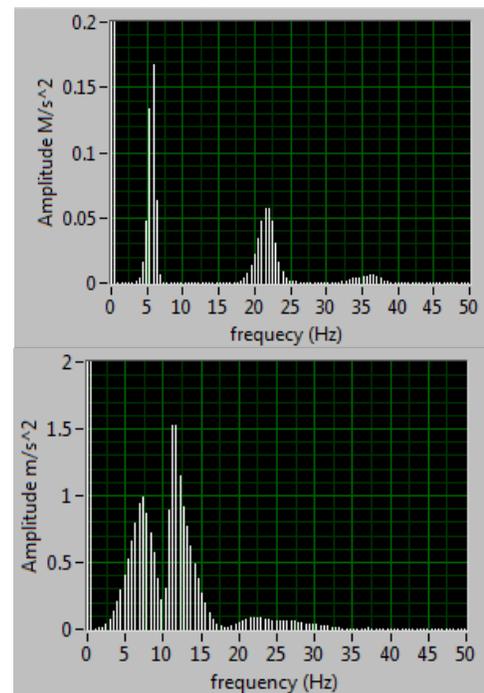

Figure 9 (top): A Fourier transformation of the original model.
Figure 10 (bottom): A Fourier transformation of the model damped with larger cross-bracing.

To get the best idea of how the sharpness of the resonance was being affected by the different types of damping, we looked at the Fourier transformation of each system with the same impulse applied at the same point and used the graph to calculate Q using the resonant frequency and the full width at half of the amplitude. The natural frequency was interpreted as the frequency at which the peak of each spike occurred. The mass damped model had a 243 ± 1 gram strip of plywood centrally place on the top floor. The spring damped system consisted of eight springs and four masses placed in each corner of the top floor. The total mass added to the system was 42.8 ± 0.1 gram in this case. Cross beams were added to each face of the upper part of the building in the shape of an X for the cross-brace damped system. The results show that we were able to achieve our goal of reducing the Q-value in all three of the basic damping techniques that we applied. Cross-bracing was the most effective damping technique at lowing the Q-value. The other two damping techniques produced the same results. One thing to note about these two techniques is the difference in their masses. The mass placed on the top floor was much larger than the masses used in the spring damping. We would need to look deeper into this type of damping to fully understand how much more effective this technique could be but from our results, it appears that it is more efficient that placing a large mass on top of a building.

Model	w_0 (Hz)	Δw (Hz)	Q
Original	6	0.50	12
Mass Damped	6	1.1	5.5
Cross-brace Damped	7	3.0	2.3
Spring Damped	6	1.1	5.5

Table 1: Sharpness of resonance results from the four models that were tested.

After applying two different types of cross-bracing to the CAD model, we performed finite element analysis again to see how the modes were affected by the damping technique. The first change was adding smaller cross braces to the corners of each floor of the building. The first three modes acted similar to the original design but the extreme displacement sections seemed to be smaller. These modes were also excited at slightly higher frequencies: 4.04 Hertz, 4.59 Hertz, and 5.48 Hertz in ascending order. The second cross-bracing technique applied included diagonal beams that formed a large X on each face of the upper section of the building. The modal analysis of this technique proved to look similar to the original design but the modes of this design were excited at even higher frequencies: 4.49 Hertz, 6.24 Hertz, and 7.52 Hertz in ascending order.

Throughout the experiment, we used both a physical model as well as a CAD model to test where resonant frequencies were and what modes would be excited at those frequencies. We also used a relationship to scale the frequencies of the full scale model to the scaled model
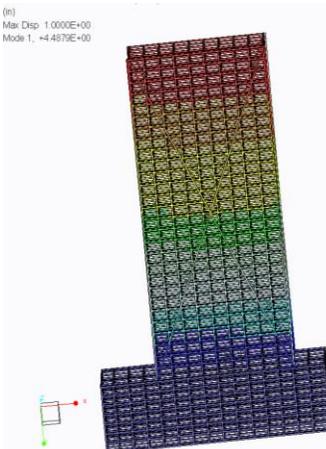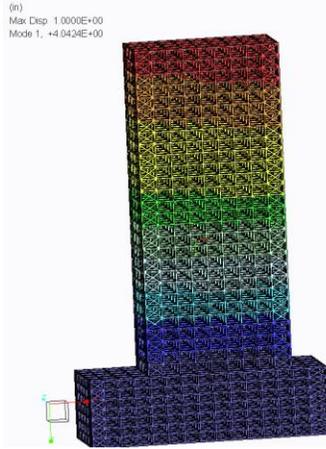
Figure 11 (top): Mode 1 of the first cross bracing technique.
Figure 12 (bottom): Mode 1 of the second cross-bracing technique.

using Young's modulus, the mass density, and the height of the materials. The results to these tests did not agree and this could be due to a few main reasons. First, we used hot glue to fasten the joints together in the physical model where the CAD model was entirely made of steel. Hot glue has completely different material properties than balsa wood. Next, the balsa wood that was used, did not have a uniform density or stiffness throughout every single piece that was used. In our calculations, we assumed uniform material properties throughout the entire structure. Finally, the measurement and construction precision was not very good which can be seen as the physical model has many imperfections. To improve these flaws, we could have taken more time in creating a better detailed CAD model as well as a more precise physical model. We could also could have also used different building materials that have more uniform physical properties.

Conclusion

Resonance occurs when oscillations of a system match the natural frequency of that system. When a system is vibrating at a resonant frequency, the system responds at much greater amplitudes. In buildings, this could be violent swaying which could lead to severe structural damage or even complete collapse. An effective way of determining how susceptible a building is to this catastrophic shaking is the Q-value. The main goal of architects and engineers is to make building as safe as possible. This means that when designing a building, architects and engineers need to consider the effects that wind, earthquakes, and the people inside have on a building's Q-value. In measuring the Q-value of the original model and several variations of that model with damping, we found that adding cross-bracing was the most effective way to lower the Q-value of the model.

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