Logarithms

A **logarithm** is used to solve an exponential equation, where the unknown is the exponent. Logs and bases are inverse operations, like multiplication and division or exponents and roots.

$\boldsymbol{y = b^x}$ can be rewritten as $\boldsymbol{\log_b y = x}$

So, when you compute log 3, you are actually solving for x in $10^x = 3$.

For example, to solve for x in $10^x = 25$, rewrite using log:

$10^x = 25$	b is 10, y is 25, and x is our unknown exponent
$\log_{10} 25 = x$	the log button on a calculator is $\log_{10}$, so push 'log 25'
log 25 = 1.398	
1.398 = x	

To check your answer, put 1.398 back into the original problem: $10^{1.398} \approx 25$

Sometimes you have a base other than 10. When that is the case, you can use the conversion formula given below to change bases.

$$\log_b x = \frac{\log x}{\log b} = \frac{\ln x}{\ln b}$$

$$\log_2 8 = \frac{\log 8}{\log 2} = 3$$

ln is called the natural log; it is actually $\log_e$. e is like pi, in that it is a symbol for a commonly-used number that doesn't repeat or terminate. $e \approx 2.718$

Logarithms are used to solve pH problems.

$pH = -\log [H^+]$

If you are given $[H^+]$, simply plug the value into the equation and evaluate to find the pH.

Find pH when $[H^+] = 1.2 \times 10^{-5}$

$pH = -\log[1.2 \times 10^{-5}]$

$pH = -(-4.921)$

$pH = 4.921$

If you are given pH, use inverse operations to solve for $[H^+]$.

Find [H+] when pH = 3.8.

$3.8 = -\log [H^+]$	divide by -1 to move the negative sign
$-3.8 = \log [H^+]$	10^ is the inverse operation of log
$10^{-3.8} = [H^+]$	evaluate to simplify
$1.585 \times 10^{-4} = [H^+]$	

Because logs are related to exponents, there are some rules similar to exponent rules that apply to logarithms (with any base, including *e*). These rules are helpful when using formulae that include logs.

$\log A + \log B = \log (A \times B)$

$\log x^n = n \log x$

$\log_b \left(\frac{1}{t}\right) = -\log_b t$

$\log A - \log B = \log \left(\frac{A}{B}\right)$

$\log_b 1 = 0$