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PHY 312 - Sherman
Negative Index of Refraction

Introduction:
Throughout its history humanity has always created models of our world in an attempt to understand and predict it. These models have served us reasonably well and have enabled us often to predict events based upon our past experiences of them. However, a recently new occurrence is that our models have grown so complex that they sometimes predict the existence and behavior of phenomenon as of yet entirely unknown to us. Perhaps even more spectacularly, they are often correct. One instance of this is Victor Veselago's prediction, in 1967, of the characteristic behaviors of a material with negative electric permittivity (ε) and magnetic permeability (μ). He called these Left Handed Materials.

Theory:
If we take Maxwell's equations in matter:

$$\nabla \times \mathbf{E} = -i\omega\mu\mathbf{H},$$

$$\nabla \times \mathbf{H} = i\omega\varepsilon\mathbf{E},$$

and plug in the equations for plane waves:

$$\mathbf{E} = \mathbf{E}_0 \exp(-i\mathbf{k}\cdot\mathbf{r} + i\omega t)$$
$$\mathbf{H} = \mathbf{H}_0 \exp(-i\mathbf{k}\cdot\mathbf{r} + i\omega t)$$

And simplify, then the equations become:

$$\mathbf{k} \times \mathbf{E} = \omega\mu\mathbf{H},$$

$$\mathbf{k} \times \mathbf{H} = -\omega\varepsilon\mathbf{E}.$$

In this case $\varepsilon=\varepsilon_r\varepsilon_0$, and $\mu=\mu_r\mu_0$, where ε_0 is the permittivity of free space and μ_0 is the permeability of free space. From these equations one can plainly see that for positive values of ε and μ the direction of wave propagation can be obtained by using the right hand rule with first the electric and then magnetic fields. That is to say that **E**, **H**, and **k**, form a right handed system of vectors. If, however, we have negative values for ε and μ then the above equations experience a sign change and may be rewritten as:

$$\mathbf{k} \times \mathbf{E} = -\omega|\mu|\mathbf{H},$$

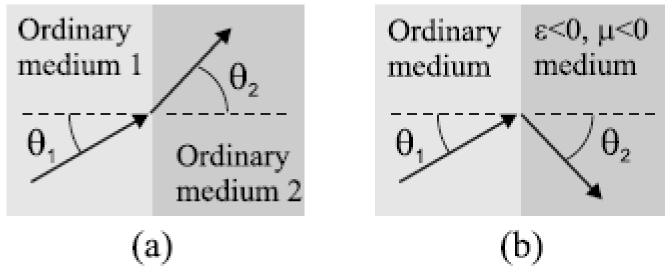
$$\mathbf{k} \times \mathbf{H} = \omega|\varepsilon|\mathbf{E},$$

In this case the right hand rule applied in the way mentioned above points us in the direction opposite to that of the wave propagation. For this reason we say that **E**, **H**, and **k** form a left handed system of vectors for a material with negative ε and μ. Thus the term Left Handed Material.

Another implication of a material with negative values for ε and μ is the unusual refraction between them and normal materials. If we examine Snell's Law:

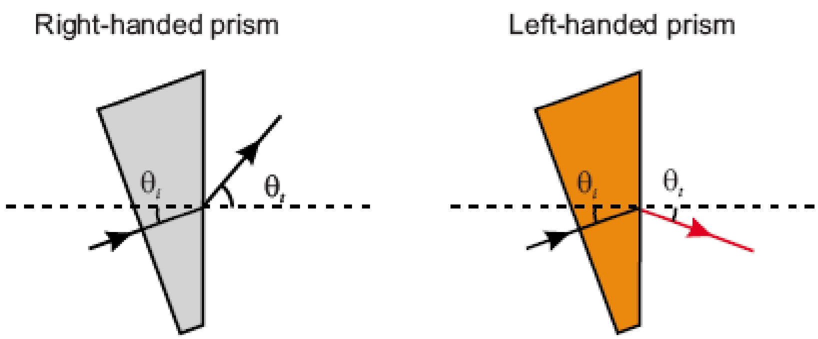
$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

it becomes readily apparent that if the wave velocity in one material is negative, then so too is its index of refraction. Thus the angles of incidence and refraction must, for a wave traveling in between a left handed and right handed material, have opposite signs. The wave will then experience a negative diffraction as opposed to a similar wave traveling in between two like media:



Refraction at the boundary between (a) two ordinary media and (b) an ordinary medium and a left handed medium.

Methodology:
The easiest way to demonstrate refraction in a material is to cut a prism from it, send a beam through it, and then measure the angle of diffraction. In right handed materials the waves will refract normally, in left handed materials, having a negative index of refraction, the waves will have a negative angle of refraction:



We shall attempt to construct a metamaterial (a material engineered to have properties not typically found in nature due to its unusual structure) that has a negative index of refraction at microwave frequencies. In order to accomplish this ε and μ must both be negative at the appropriate frequencies. We can make this task significantly easier by taking advantage of the fact that we will be using linearly polarized plane waves, for, in this case, it is only necessary to make a material that has a negative ε in the direction of the electric field **E** and negative μ in the direction of the magnetic field **H**.

The way in which we will achieve a negative value for ε is by stacking metal sheets parallel to the direction of **E**. In this way each pair of sheets will act as a parallel plate wave guide. The propagation constant of the fundamental transverse electric mode in a parallel plate waveguide is:

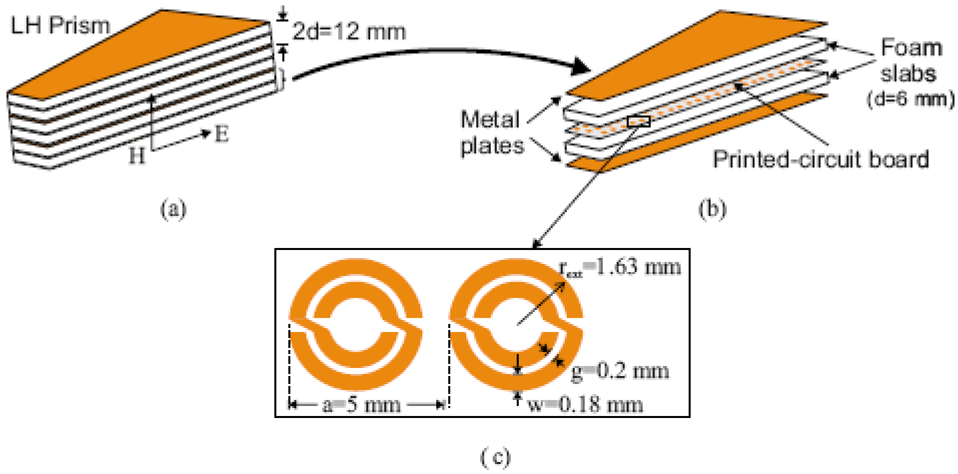
$$\beta = \sqrt{k_0^2 - \left(\frac{\pi}{d}\right)^2},$$

where k_0 is the free-space wave number, d is the distance between the sheets, and π/d is the cutoff wave number. So the effective relative permittivity of such a waveguide is:

$$\varepsilon_r = \left(\frac{\beta}{k_0}\right)^2 = 1 - \left(\frac{\pi}{dk_0}\right)^2 = 1 - \left(\frac{c}{2df}\right)^2,$$

where f is the operating frequency, c is the speed of light in a vacuum, and $c/2d$ is the cutoff frequency. From this it is clear that, by selecting an operating frequency less than the cutoff frequency, we can obtain a negative value for ε.

To get a negative value for μ we will make a planar array of split ring resonators that will be placed between the metal sheets perpendicular to the **H** field. This results in a magnetic flux passing through the rings which in turn induces currents in the rings causing them to produce their own magnetic field. At frequencies just above the resonant frequency of the rings this causes μ to become negative.



(a)Multilayer structure of the fabricated prism of left handed material. (b) Details Of the structure of a layer consisting of printed circuit board with resonant rings Between a pair of foam slabs; the layer is covered by metal plates. (c) Geometrical Details of the resonant rings
(Dimensions are not necessarily accurate)

Fabrication and Setup:
The first step in prism fabrication is to create the split ring resonator sheets. Then we must cut equally sized sheets out of a conductive material and foam. We then stack these together in the pattern demonstrated above and make a 20º cut to create our prism.

To make the split ring resonator sheets we used a technique for making printed circuit boards. We first created the design for our resonators on the vector graphics software, Inkscape. Once we were satisfied with the dimensions of our design we printed it onto high gloss photo paper. We then thoroughly polished sheets of copper using scouring pads until they were completely blemish free, and then wiped them down with either acetone or alcohol to remove any residual oils or imperfections that might interfere with the printing process. We taped the copper to the high gloss paper so that the polished side was in direct contact with the SRR design, being careful not to get any fingerprints or other contaminants in between the copper and the paper. We preheated the iron to the third hottest setting and prepared to iron the design from the paper onto the copper.

The ironing procedure was as follows:

First, placing a blank sheet of paper in between the copper and the iron, gently press iron to copper and preheat for 30 seconds.

Second, flip copper over so that the high gloss paper is on top. Then, placing a blank piece of paper in between the high gloss paper and iron, press down firmly and iron the top edge, rotating iron in small circles for 50 seconds.

Third, rotate paper 90 degrees and again iron top edge for 50 seconds.

Fourth, repeat the third step twice more until all edges have been ironed.

Fifth, remove blank sheet of paper and place copper and high gloss paper in a bath of water until the tape separates. Then remove high gloss paper from copper sheet.

After the ironing, with a little luck, the toner would have transferred to the copper leaving us with a copper sheet with the SRR design printed on it in toner. This is due to toner being basically made of plastic and so the heat and pressure from the iron melted the toner, transferring it to the copper sheet.

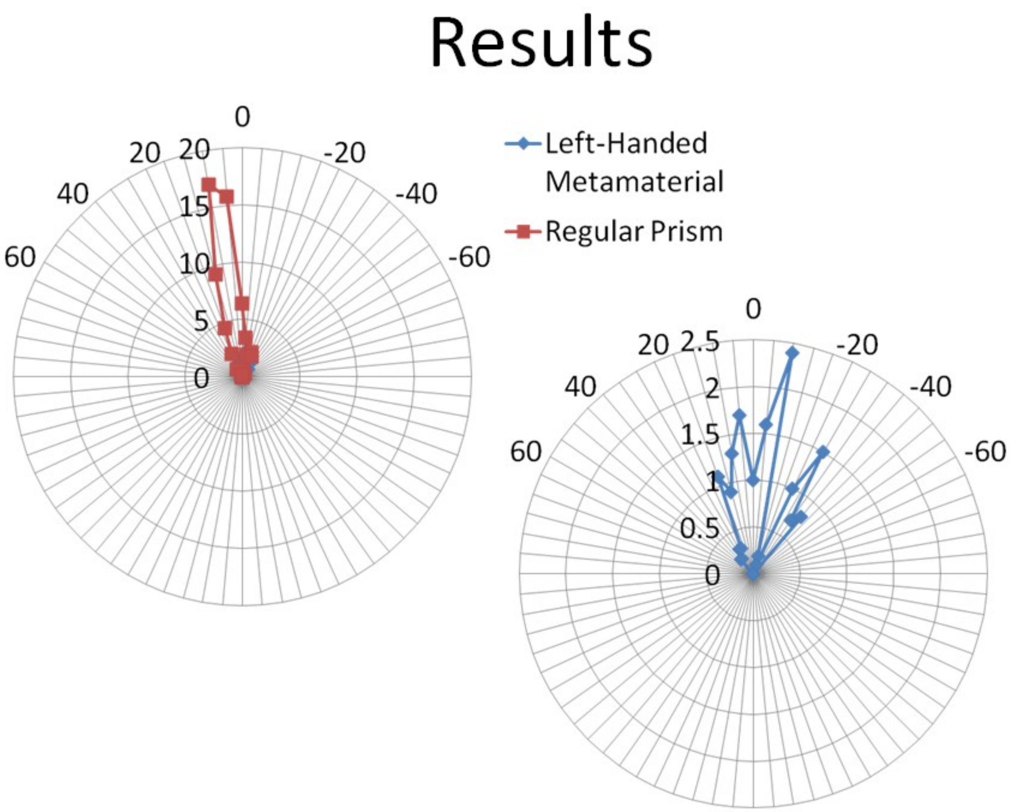
Once we had a sheet with toner transfer of sufficient quality, we wanted to remove the copper that was uncovered by the toner, leaving us with a sheet covered in copper SRRs. To do this we submerged the copper sheets in an etchant, perturbing it constantly with an air pump, until all of the visible copper was stripped away. We began by using a mix of one part concentrated (~30%) hydrochloric acid to two parts hydrogen peroxide (3%). This etchant worked reasonably well for the first few boards, but eventually slowed to a standstill. We tried several things to remedy the problem including the use of an air pump to force air into the solution and adding a small amount of hydrogen peroxide, but the solution remained ineffective. We therefore decided to switch to using ferric chloride as our etchant. This worked better but required constant perturbing to etch in a timely fashion and required replacement every four to six boards. The etching time varied but usually took between 1.5 and 2.5 hours.

With a sufficient number of SRR sheets made, we were able to make our material. For the conducting plates we used either aluminum foil or copper sheets similar to those on which we printed the SRRs. For the spacing material we used poster board which was conveniently already 6mm in width and therefore needed only to be cut into the proper shape. We tried several different configurations and sizes for the prism, but the most successful was the setup diagrammed above with three sheets of SRRs and five conducting plates alternating in between sheets of foam.

We inserted this into a mounting base that we constructed out of wood and covered in aluminum foil to provide shielding. We also added additional sheets of copper around the base to provide further shielding. To measure the angles of refraction we attached a rotating arm to a protractor and placed the microwave receiver on it. The microwave transmitter was positioned so that it was facing the prism at a normal angle. Both the transmitter and receiver were around 30cm away from the prism.

Procedure and Results:

In order to make our measurements we rotated the arm with attached receiver from 40º to -40º, with 0º being aligned with the transmitter, making measurements every 5º. For comparison purposes we also constructed a plastic prism which would behave like a regular material. Our results for the measurements of both prisms are:



The regular prism clearly has one strong transmission lobe around 10º, which is about 30º offset from the normal. This seems to demonstrate that our apparatus for measurement works well and our mounting base appears to be sufficiently shielded.

The transmission for our left-handed material is clearly more erratic than that of the regular one, seeming to have several major lobes of transmission. The first lobe, centered around 5º, implies that some of our signal is experiencing normal refraction through the material. The second lobe, centered around -10º, seems to show part of the signal experiencing an index of refraction less than one. The third lobe, centered around -30º, shows that part of the signal is experiencing a negative index of refraction in our material.

Our prism, while obviously imperfect, did manage to demonstrate at least a partial negative index of refraction. The other lobes are most likely due to imperfections in the fabrication process, most specifically the printing of the SRR sheets. The ironing process, although greatly improved throughout the course, still inevitably resulted in some rings being squashed or otherwise not completely transferred. This likely resulted in areas of the boards with slightly different magnetic responses than expected, which would account for the undesirable transmission lobes. It should be noted that we would expect to see some transmission at about 0º due to the fact that our prism is neither infinite nor continuous and thus does not entirely satisfy the Ewald-Oseen extinction theorem. While we do get some signal in this area it is not a major lobe.

We conducted one final test of each component of the prism separately to try and more definitively determine the source of our problem. We first measured the signal directly through only the foam that we had used to space our SRR and conductive sheets. As expected this had no effect. Second we used the above prism setup but removed only the SRR sheets. This entirely attenuated the signal, which is what we would expect with a negative electric permittivity and positive magnetic permeability. Next we tested the prism with only the SRR sheets. In this configuration around half of the signal was lost, however we would hope that it would be entirely attenuated as in the case above. This at least confirms that the problem does in fact lie with the SRR sheets.

Future Plans:

Although our attempt to create a material with a negative index of refraction was not entirely successful there are several steps we can take to attempt to improve upon our work. The first is fabricating the SRRs with a more precise method such as readily available photoetching methods for creating PCBs. The second is the development of a theoretical model for the behavior of our SRRs. This would allow us to tune them much more precisely to the appropriate frequency. Conversely it would also be helpful to acquire a tunable microwave emitter, which would make a theoretical solution unnecessary.

Resources:

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K. Aydin, I. Bulu, K. Guven, M. Kafesaki, C. M. Soukoulis and E. Ozbay, *Investigation of magnetic resonances for different split-ring resonator parameters and designs*, New Journal of Physics 7 (2005).
V. C. Ballenegger, T. A. Weber, *Ewald-Oseen extinction theorem and extinction lengths,* Am. J. Phys., Vol. 67, No. 7 (1999).