Optical Simulation of Quantum Logic gates

Background

Today's digital computers are built out of circuits that have definite, discrete states: on or off, zero or one, high voltage or low voltage. Quantum computing systems are different from classical ones because they utilize *superposition* states in between the classical, discrete states. One of the earliest scientists who looked into the possibility of quantum computers was the famous physicist Richard P. Feynman who asked what computing system could contribute to the understanding of quantum physics. As a classical computer cannot simulate a quantum system, Feynman suggested that a quantum computer might be able to handle it. This might seem as a purely theoretical instrument in physics and in fact, in earlier days, scientists thought that there is nothing special in terms of computational algorithms a quantum computer could do that a classical computer could not.

However, it turns out that a program operated by a quantum computer can factor large numbers faster than any known algorithm for a classical machine. The quantum factoring algorithm, as it utilizes interference effects of different states,allows the computer to explore all possible solutions to a problem at the same time. One of the "hard" problems of mathematics, known as factorization problem, is the foundation of many of today's methods of encryption such as RSA used in electronics bank accounts. If quantum factoring algorithms are available, such encryption methods will become insecure. In this paper, I will mention how we could simulate certain quantum logic gates using only linear optical elements, how I construct some of those optical elements and, also explain what quantum systems we could build using those optical analogues of quantum logic gates.

Before we learn about quantum logic and its logic gates, let us first remember the nature of classical gates. One of the simplest classical gate, the NOT gate is wholly deterministic: Once the input is known, the output is determined with absolute certainty. If the input is zero, the output is one and

vice versa. Let's say we make a logic gate that completely randomizes its input, producing a 0 or 1 output with equal probabilities. The transformation matrix for this function is:

$$\begin{array}{c|cc} & 0 & 1 \\ \hline 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & \frac{1}{2} & \frac{1}{2} \end{array}$$

This probabilistic version of NOT gate models a fair coin flip. Both the ordinary Boolean NOT gate and the probabilistic versions of it are still constructions of classical physics. A quantum-mechanical gate is far stranger. Whereas classical bits have the value 0 or 1 at all moments, quantum bits, called qubits, can occupy a "superposition" of the 0 and 1 states. It would be a mistake to say that the qubit has an intermediate value between 0 and 1.Rather, the qubit is in both the 0 state and the 1 state at the same time, to varying extents. When the state of the qubit is eventually observed or measured, the "wavefunction" collapsed and the qubit occupies a definite state of either 0 or 1. Such a quantum

$$\begin{array}{c|cc} & |0\rangle & |1\rangle \\ \hline |0\rangle & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ |1\rangle & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{array}$$

logic gate is known as Hadamard gate and is one of the most important quantum logic gates. The mathematical expression for a Hadamard gate matrix is shown below.

All quantum logic gates must be reversible, which means if the outputs of an operation are put through the machine in the reverse direction, the original inputs have to be recovered, Classical AND and OR gates do not obey this rule as we inevitably lose information when two input bits are reduced to a single output bit. Therefore, we do not have AND gates or OR gates in quantum logic. However, some classical gates such as NOT, CNOT gates do exist in quantum logic but their effects when combined with a "genuine" quantum logic gate such as the Hadamard gate mentioned above become quite remarkable and distinct from what we would expect from a classical setup.

Experimental setup

In 1998, N.J.Cerf et al. from California Institute of Technology proposed a method for simulating small-scale quantum circuits only using traditional, linear optical devices including beam splitters, phase shifters and polarization rotators. An optical symmetric beam splitter is known to act as a quantum square root of NOT gate or Hadamard gate since the output ports are indeed in a superposition state between 0 and 1 if we take one input port as the vacuum state 0 and the second one as a single-photon state 1. Similarly, phase-shifters (quarter or half-waveplates) can be used as quantum phase-shifting gates acting on one mode of the photon. Cerf and co. suggested that using nine beam splitters (seven symmetric beam splitters and two polarizing beam splitters), six quarter-wave-plates, two polarization rotators and a number of mirrors/prisms, we could build a three-qubit quantum circuit which will allow us simulate quantum entanglement and teleportation. Before we explore the entire circuit, I will explain how these optical pieces function, how I "construct" some of them and how I verify as their properties experimentally.

First of all, 50-50 beam splitters, on a macroscopic level, will reflect 50 percent of incident light and will allow the other half pass through it. If we look at the individual photons however, they would be in a superposition state between two different routes. The beam splitter, combined with the two 90-degree phase-shifters at the input and output with their angles 45 degree tilted, will act exactly as a Hadamard gate on the location qubit. Beam splitter glasses require proper mounting and adjustment.

Making polarization rotators is fairly straightforward. Corn syrup is one of the materials which interact with the polarization of light. (it rotates the light different degrees depending on its wavelength. Blue light will rotate more than red light, because it has a shorter wavelength.) A lot of other proteins have this similar property. Since polarization of the red laser should be rotated from

horizontal to vertical, the depth of corn syrup required for 90 degree rotation is measured first. Tubes of the required depth are filled with corn syrup and then can be used as 90-degree polarization rotators, which will be acting as optical versions of quantum CNOT gates.

Polarization beam splitters were readily available and were verified to act as CNOT gates. These split the unpolarized light into two different directions as horizontally polarized and vertically polarized light. If the light is already polarized, it goes in the direction in accordance with its polarization state.

I attempted to replicate the quantum circuit with eight quantum gates shown in the paper by Cerf et al. However, it was discovered that in order to build a working Hadamard gate with beam splitters and phase-shifters, it requires extremely precise adjustments (equipments also required for those adjustments) and at the point of writing this paper, I have not built a working Hadamard gate. However, we observed the interference fringes from the two beam splitters when they are set up as in a Mach-Zehnder optical interferometer.

Although I could not replicate the entire circuit, it would be noteworthy to briefly mention and explain the setup of the circuit, and also, interpret what the quantum implications of this circuit are. First, two location qubits and four different "paths" (00,01,10,11) are created as a single photon pass through two Hadamard gates with the initial state of the first qubit set as 0. Among these four paths, only on path 01 or 10, the polarization state of the photon is flipped with the use of two polarization rotators. Two more Hadamard gates were used, one for 00 and 10, and one for 01 and 11. Therefore, only the first location qubit is affected by the Hadamard transformation in those two gates. Furthermore, two polarizing beam splitters were placed with the same value of the first qubit(i.e. 00 and 01 as one, 10 and 11 as the other) passing through each of them. Two last beam splitters as Hadamard gates were placed at the outputs of the polarizing beam splitters.

In quantum logic interpretation, the above mentioned circuit performs as a teleportation device for the quantum qubits. Regardless of the fact that there are eight possible outcomes (four paths and two polarization states), the photon will always reach to two output states (specifically 10 and 00). What this means is that the initial state of the first qubit is now teleported to the second qubit once the latter is observed or measured. It should also be mentioned that the photon's location qubits were also entangled with its polarization qubit, which means observing polarization state will instantly make the location qubits take the appropriate values.

Conclusion

Making Hadamard gates using beam splitters is not as straightforward as I expected. However, other equipments that I acquire were verified to have the property of acting as quantum gates on the qubits. This setup, however, is only a simple optical simulation of a small number of quantum logic gates, and it requires exponentially increasing optical equipments for increasing qubits. Therefore, it is not practical to build circuits with more than 4-6 qubits in this configuration. Previously it was believed that it was impossible to build an optical setup without exponential increase in the equipments, but KLM protocol, proposed in 2000, eliminates such a requirement.

So far, I have mentioned how we could build optical versions of quantum logic gates using only linear optical equipments. Quantum computing systems with lots of such gates will do parallel computations at the same time. Destructive interference would eliminate those processes that were not of interest while constructive interference would enhance those met the required conditions. Current scientific community has taken a profound interest in the development of quantum optics which has been growing since the 1980s, and in fact, Wineland and Haroche, the two Nobel Laureates in physics, earned their merit with their work in quantum optics and computing systems. Quantum computers would probably take years to come, but their advent would radically change our everyday

lives and our understanding of the natural world.

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