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Demonstrating Special Relativity Through the Detection of Cosmic Ray Muons

**Introduction**

The purpose of this experiment is to gain insight into special relativity and how it affects particles encountered on Earth by attempting to replicate early experiments used to prove relativity. Special relativity explains how moving particles experience time differently than non-moving particles. An observer in one frame will have a different version of events than an observer in a second inertial frame that is moving relative to the first. In other words, simultaneity is not consistent between inertial frames. One consequence is that time passes slower in a moving frame compared to one at rest. The equation relating to time is[1]:

$$\Delta t = \gamma \Delta t_0$$

The Lorentz factor γ is equal to[1]:

$$\gamma = \frac{1}{\sqrt{1 - (\frac{v}{c})^2}}$$

where v is the speed of the moving object and c is the speed of light ($2.998 \times 10^8$m/s).

To demonstrate time dilation, we can consider cosmic ray muons. Muons are created in the atmosphere at altitudes around 15km when cosmic ray particles (mainly protons and alpha particles) collide with air molecules, creating pions and other particles that then decay into muons[2,3]. A picture of this process is shown below[3].
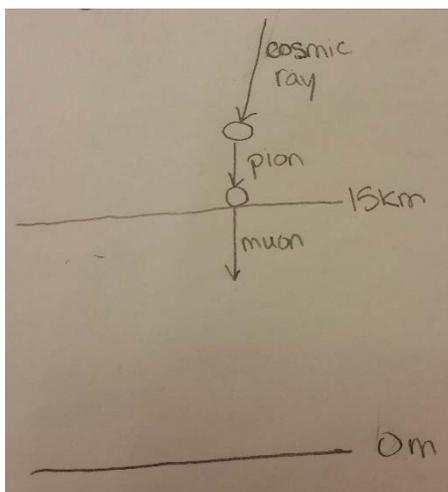


Figure 1: High above the Earth, cosmic rays create pions which then decay into muons

Moving at speeds near the speed of light, muons are relativistic particles.  They have an average lifetime of 2.2µs in their rest frame[4], but, due to relativity, they last much longer than this according to Earth's time.  The goal of this project is to observe this phenomenon.

To observe this effect, it is necessary to observe muon counts at various altitudes.  Newtonian physics predicts that muons traveling at c for 2.2µs will travel 660m, though they decay at an exponential rate, so this is not an exact number.  We can use an exponential decay formula to predict the fraction of muons that survive to various altitudes[5].

$$\frac{N}{N_0} = e^{-(\frac{T}{t})}$$

T is the time it takes to travel to the altitude of interest (calculated by $T = c/(15km - Altitude)$), and t is the muon lifetime.  If relativity is not taken into account, then we use 2.2µs as the muon lifetime, and the formula predicts that almost all muons decay far above the Earth, within a couple kilometers of being created.  However, applying relativity yields a different result.  Cosmic ray muons have been measured with an average speed[6] of 0.994c-0.998c.  Using 0.996c yields γ = 11.2, corresponding to a muon lifetime in Earth's frame of 24.6µs.  Substituting this value for t results in noticeable fractions of muons reaching ground level.  The table below shows the fraction of muons predicted to survive to various altitudes.  The amount of muons increases[3] about 10% for each gain in altitude of 1000ft.  A picture showing this principle is shown below (not accurate ratios).  If we can measure muon counts at various altitudes and get results similar to those below, then that is evidence of relativity.

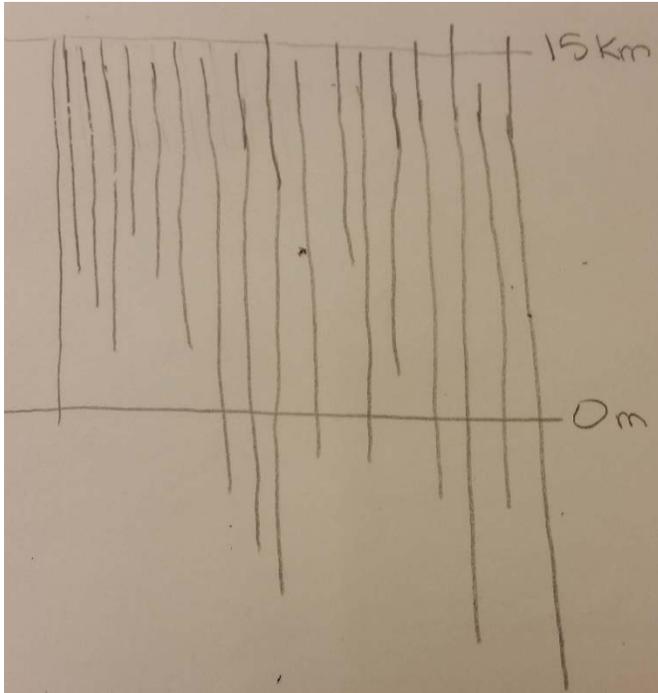| Altitude (km) | Distance (km) | Time (µs) | Survival Fraction |
|---|---|---|---|
| 15.000 | 0.000 | 0.000 | 1.000 |
| 12.000 | 3.000 | 10.000 | 0.666 |
| 10.000 | 5.000 | 16.667 | 0.508 |
| 8.000 | 7.000 | 23.333 | 0.387 |
| 6.000 | 9.000 | 30.000 | 0.295 |
| 5.000 | 10.000 | 33.333 | 0.258 |
| 4.000 | 11.000 | 36.667 | 0.225 |
| 3.000 | 12.000 | 40.000 | 0.197 |
| 2.000 | 13.000 | 43.333 | 0.172 |
| 1.000 | 14.000 | 46.667 | 0.150 |
| 0.000 | 15.000 | 50.000 | 0.131 |

Figure 2: Muons created around 15km decay exponentially.  A fraction of muons reach the ground, while most (87%) decay somewhere in the atmosphere

**Detector**

The main work in this experiment is designing and building a muon detector that can be transported and used at various altitudes.  This means it must be relatively small and able to be powered by a small portable power supply like a battery.  The whole design can be broken down into three parts:  high voltage supply, Geiger tube array and circuitry, and coincidence detection.  The general arrangement of these components is shown in Figure 3.
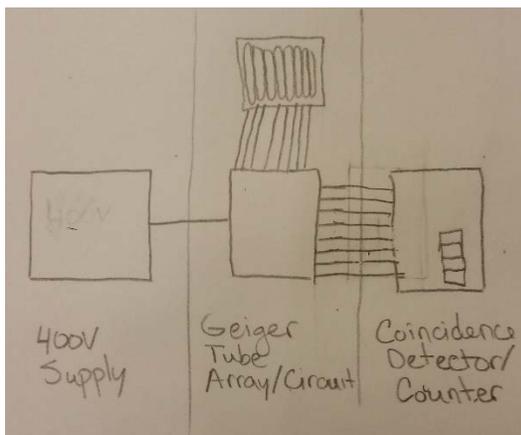


Figure 3:  400V supplied to the Geiger tubes, which feed to the coincidence detector/counter

**High Voltage Supply Circuit**

The Geiger tubes used in this project need 400V of direct current (DC). This can be generated with high voltage supplies that can be powered by wall outlets, but this device must be portable, so we needed a small circuit able to convert a small voltage into a much larger voltage. The original goal was to use a 555 timer chip and a 5239 chip to provide a +5V/-5V square wave to a transformer, which would amplify the signal and output a sine wave of alternating current (AC), which could then be put through a rectifier to convert it to DC. The outputs at each stage are shown in Figure 4. After trying several combinations of input voltages and voltage multipliers to most closely get 400V, we decided that there was too much ripple in the output voltage and it caused the Geiger tubes to fire at the frequency put out by the 555 chip (33.3kHz). A few years ago, another Cornell student (Kunnawalkam Raghav) also tried to build a muon detector, and his main problem was ripple in the high voltage[5].
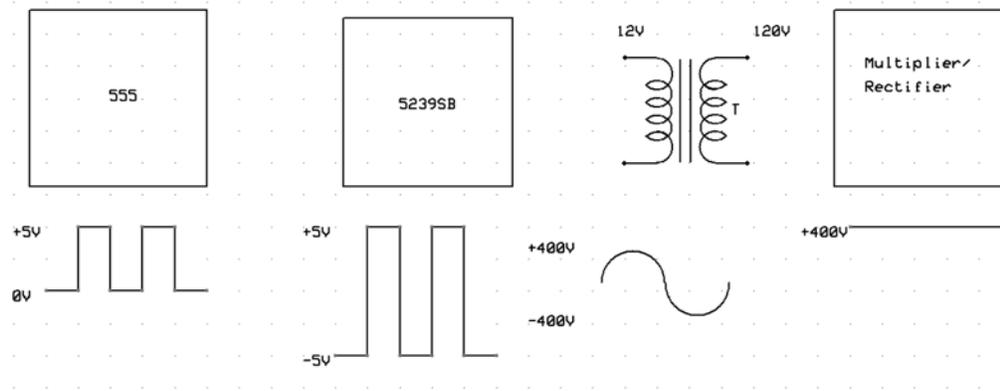


Figure 4:  Homemade 400V supply stage outputs

Instead of spending more time trying to reduce the ripple, we chose to use a 16EPC-T06 module that would save space and also put out a smoother signal without noticeable ripple. We used a LM311 and 10kΩ potentiometer (POT) as a variable input source to vary the output voltage of the module[7]. The output of the module is directed through a bridge rectifier and two 200V zener diodes in series in parallel to the Geiger tubes are used to restrict the voltage to not more than 400V. Figure 5 shows the complete circuit[7].
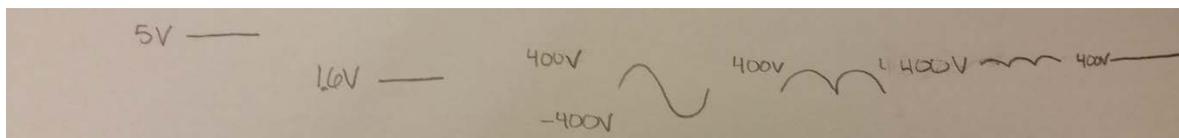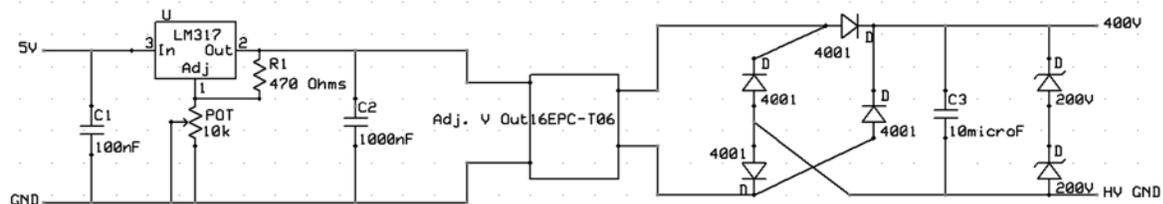
Figure 5a: 16EPC-T06 400V supply diagram, including sketches of the voltage at each stage
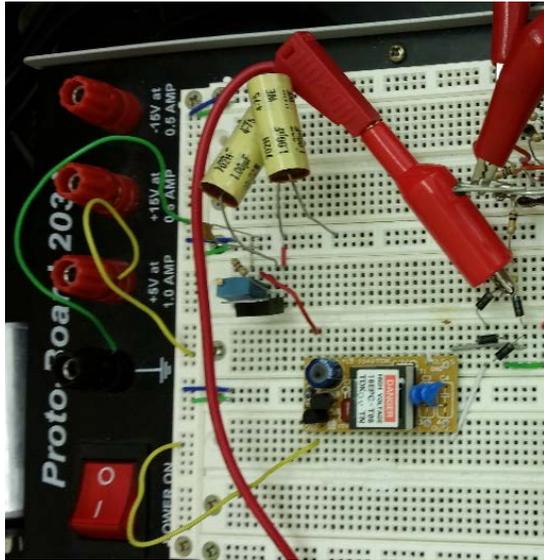


Figure 5b: 400V power supply circuit including 16EPC-T06

**Geiger Tube Array and Circuitry**

In the design we used, the muons are detected by Geiger-Müller tubes (SI-22G). Geiger tubes have a metal inner wall serving as a cathode, and a wire in the middle serving as the anode. They are filled with inert gas at low pressure. When a muon or other particle enters, the gas is ionized and current is able to flow from the wall to the wire[8]. My device consists of two rows of ten tubes each, one row on top of the other as in Figure 6. Between the two layers is a layer of shielding such as lead to block non-muons from penetrating to the second. This arrangement allows one to check to make sure that a particle has gone through two tubes as opposed to just one, which would most likely not be a muon.
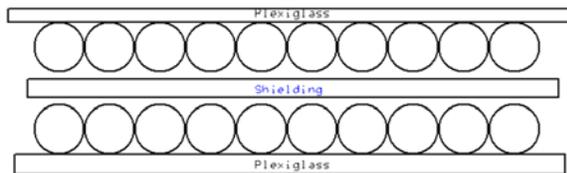


Figure 6: Two rows of ten tubes

We thought of two different ways to construct the array. One is by using a ¼" bit to drill holes the size of the Geiger tube leads (just larger than ¼") in two plexiglass sheets and suspending the tubes between them. For stability, I would drill holes in the four corners of the sheets and use 9" lengths of ¼"-20 threaded rods and nuts to secure the sheets in place. Fuse clips are attached to the leads on the outside of the plexiglass for soldering wires to, and an outer layer of plastic can be used to protect and secure the clips. Figure 7 depicts the trial of this apparatus with smaller sheets of plexiglass and three Geiger tubes. Because of the difficulty this design would introduce due to the tube connections being

on the outside of plexiglass sheets, we decided to use a different method.  In the other method, we drill two small 3/32" holes and screw the fuse clips to the plexiglass.  In this case, the Geiger tubes lay flat on the plexiglass and can be wired from the other side of the sheet.  A picture of this is shown in Figure 8.



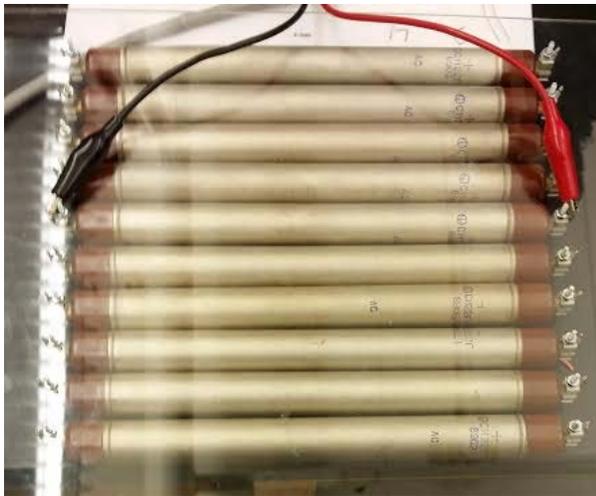Figure 7:  Three Geiger tubes suspended between plexiglass sheets



Figure 8:  Fuse clips attached to plexiglass and Geiger tube

The circuit that I am using for this project comes from this website[9], with some modifications: http://www.hardhack.org.au/geiger_muller_detector.  The modified circuit that I am using is shown in Figure 9.  When a particle enters the tube, the gas is ionized, allowing current to flow for a short time.  Thus, the voltage across the tube drops for a short pulse.  This pulse is inverted twice with a 74AC14, so the ouput from the second inverter goes low (0V) for about 1μs when a particle is detected.  In the above website's design, there are only three Geiger tubes, but we are using approximately the same design for this part of the circuit for all twenty tubes.
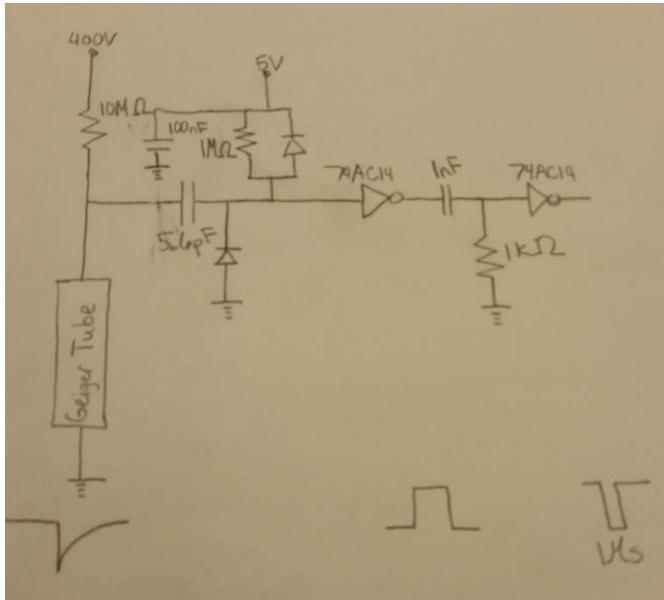
Figure 9:  Circuit for obtaining low pulses from Geiger tube detection of incident particles.  The negative spike and exponential decay across the Geiger tube (400V) is converted to a clean negative logic pulse (5V) approximately 1μs long that can be interpreted by the next piece in the circuit.

Since there are two rows of ten tubes, I am wiring two sets of circuit boards, one for the top row and one for the bottom.  As of now, I have one complete.  One board contains the 10M resistors which are then connected to the Geiger tubes and the rest of the circuit on the other board.  The layout is shown in the figures below.
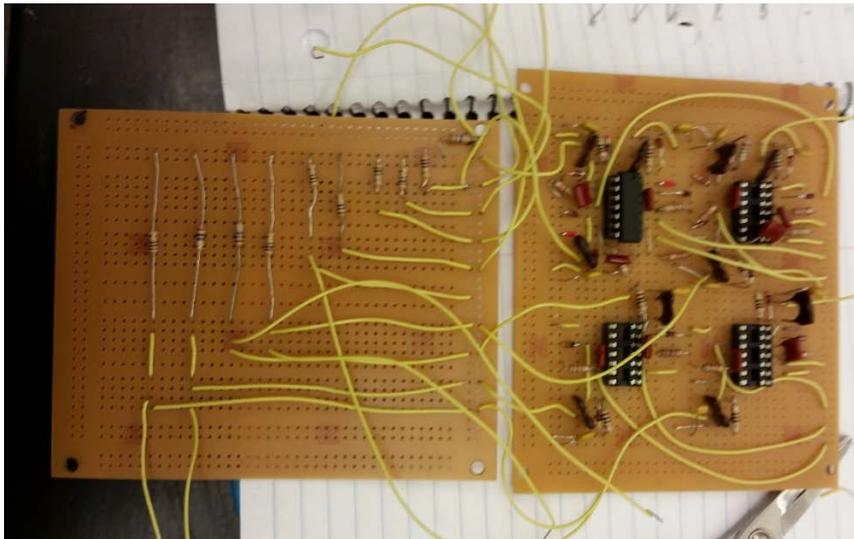


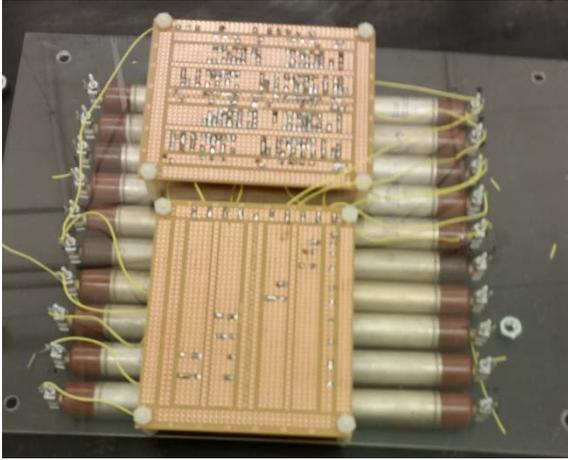Figure 10a:  Circuit board arrangement for 10 Geiger tubes

Figure 10b:  Bottom view of Geiger array and circuit



Figure 10c:  Top view



Figure 10d:  Placement of second row of Geiger tubes

**Coincidence Detector**

The final part of the detector is the coincidence detector.  This takes the output of the above circuit for all twenty tubes and determines when tubes in different rows within a set distance of each

other have detected a particle at the same time. We do this because cosmic ray muons most often hit the Earth nearly vertically[3]. Thus, a muon is likely to hit Geiger tubes that are in approximately the same position in their respective rows, as opposed to tubes on opposite ends of the row. These two situations are depicted below. The situation on the left is likely to be a cosmic ray muon, while the situation on the right may be from extraneous radiation or random discharging of the tubes at the same time. For this reason, we use the coincidence detector to count only when tubes near each other discharge simultaneously.
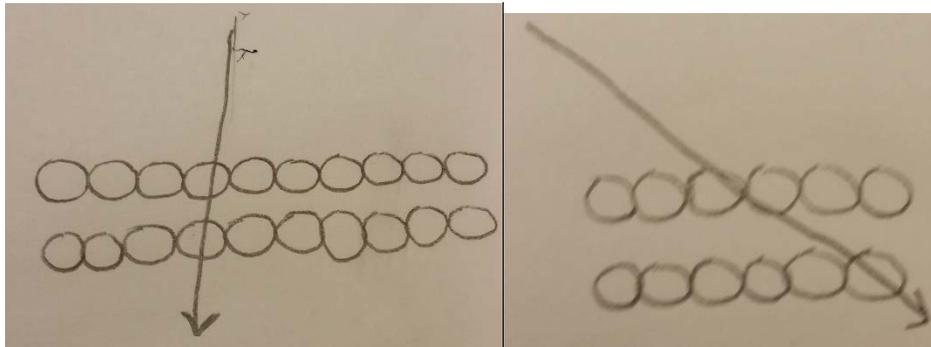


Figure 11: On the left is a vertically incident particle, most likely coming from a cosmic ray. On the right, however, the particle is much closer to horizontal, meaning it could be from background radiation that we do not want to detect.

The circuit on the hardhack website (along with other sources[10]) uses AND/OR-gates for the coincidence detector. We used this method with just two tubes to make sure that the tubes were working correctly. The outputs of the tube circuits were put into an OR gate so that that output went low when both tubes were active (LOW). When one tube was on top of the other, we could detect pulses from the OR gate, indicating that a particle had passed through both tubes at the same time. When the tubes were placed horizontally, however, no such pulses were detected. The logic gate method of coincidence detection works, but doing this with twenty tubes would require a large amount of wiring, so Professor Sherman thought of a different way that is conceptually more complex but much simpler in terms of wiring. We assign numbers to each tube in both rows, with tubes in the same position having the same value. For example, one end of the row will be tube "0," while the other end will be tube "9" (Figure 12). The top and bottom rows of Geiger tubes each feed to a set of binary encoders. When any of the lines feeding in to the encoder go low, indicating a particle event, the encoder outputs a binary value corresponding to that line/tube. If the top encoder and bottom encoder both detect events at the same time, the values are forwarded to a binary adder. The principle is that we can detect if the two events were near each other by subtracting the bottom row value (B) from the top (T) and comparing the result to how many neighbors we want to look at (e.g. only tubes directly above and below, diagonal, or two tubes down). The way to subtract using binary numbers is with the two's complement[11].

- To get the two's complement (B') we invert the bits of B and add 1. This number behaves as the negative value of B in most arithmetic
- If n is the number of neighbors that we want to check for a coincidence, we calculate T+B'+n
  - This is equal to T-B+n
- If the value is less than or equal to 2n (determined by the HC85), a coincidence has been detected and a pulse will be sent to the counting circuit (4026 LED-driver counters).
- I have also added a DIP switch so that we can control n. This will allow us to gain some information about the angle of incidence of the muons.

Examples of how the coincidence detector works are shown below, both mathematically explained and visualized in Figure 12. Figure 13 shows a schematic of how the components of the coincidence detector are connected.

# Examples

- T=0100 (4), B=1001 (9), n=1
- B'=0110+0001=0111
- T+B'+n=0100+0111+0001
    =1100=12
- 2n=2
- 12>2→ no coincidence

- T=0011 (3), B=0100 (4), n=1
- B'=1011+0001=1100
- T+B'+n=0011+1100+0001
    =0000=0
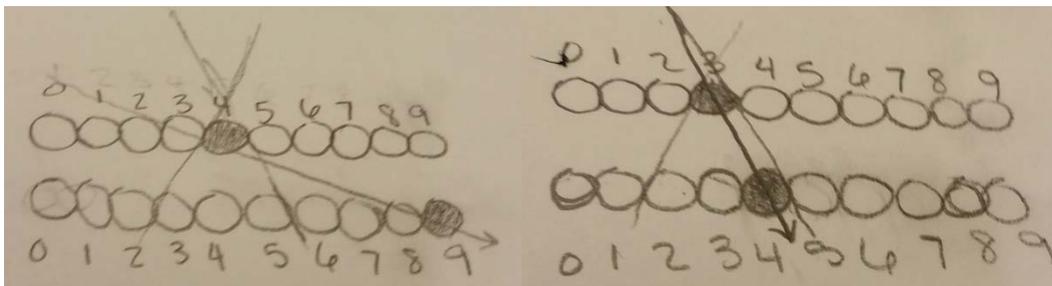- 2n=2
- 0<2→ coincidence



Figure: On the left, tube "4" in the top row fires while "9" fires in the bottom row. This is most likely not a cosmic ray muon, so the detector will not count it. On the right, top tube "3" and bottom tube "4" fire. This is a nearly vertical angle of incidence, so it is most likely caused by a muon and the detector will count it.
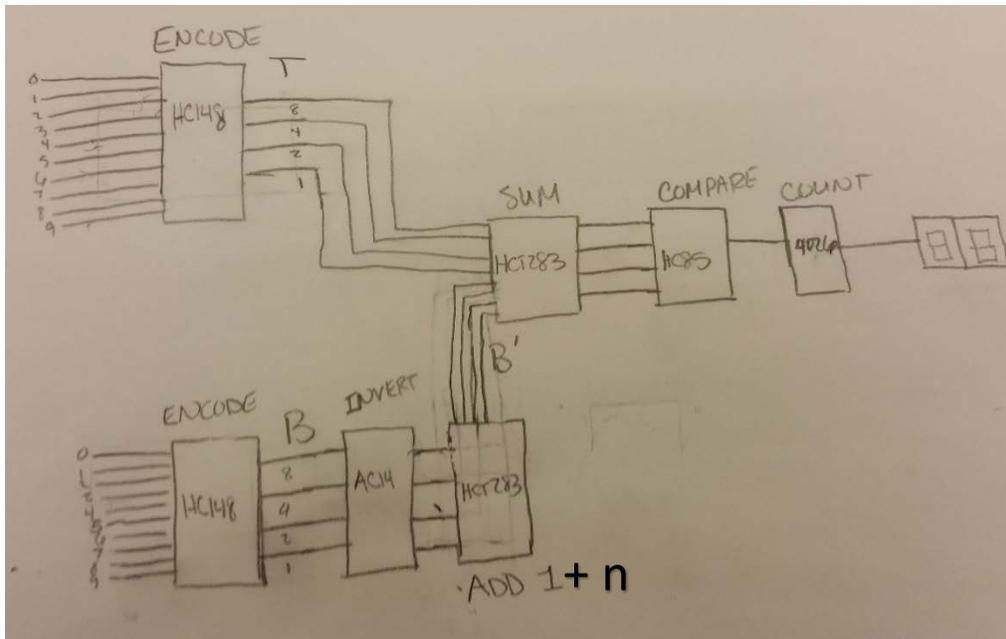
Figure 13: Coincidence detector schematic. The Geiger tube lines feed to the HC148 priority encoders. For the bottom row, the binary output value is inverted by a 74AC14 and (0001 + n) is added by an HCT283. Then the two rows are added together by another HCT283. This output value is compared to (2n) by the HC85, which sends a pulse to the counter if (2n) is larger than or equal to the value calculated.

I wired the coincidence detector circuit on a breadboard and tested it to make sure that the counter increased by one only when input lines near each other (coming from different rows) are active (LOW). Instead of connecting the twenty inputs to Geiger tubes, I connected them to data switches. The next step is to connect it to the Geiger tubes (once the circuit for the second row of tubes is complete) and start counting muons. Following that, I would attempt to power the circuits from a battery instead of the breadboard supply voltages and to make a permanent circuit (arrangement shown in Figure 14). We ordered an oversized board in order to fit the whole coincidence detector circuit on one. Ideally, we will be able to make a box to contain the detector so that it can all be moved as one piece. The plan is to make measurements at higher altitudes by flying in a small plane (courtesy of Richard Scearce), although this will not be possible until later in the year.
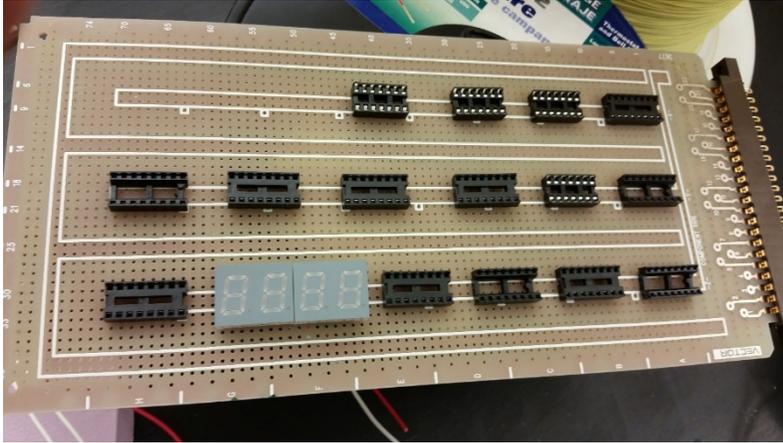
Figure 8: IC placement for permanent soldering to the protoboard

**Conclusions**

While working on this project, I learned many lessons about doing research. The circuits I am working with introduce some problems that are not encountered in most classwork. When working with high voltages, we had to use voltage dividers to measure the voltage with multimeters (DMMs) and the oscilloscope because the output impedance was too high and the scopes cannot handle high voltage. The DMM has an input impedance of 10MΩ, so we wired it in series with a 100MΩ resistor in order to get a reading 1/10 of the output voltage. The scope's input impedance is 1MΩ, so doing the same results in 1/100 of the output voltage. Other recurring problems included the wires not being shielded enough. When the Geiger tubes are wired simply with copper wires, they communicate with each other, so an event in one of the tubes causes an event in the other tube after a short delay, which can be observed on the scope. To correct this, we used coax cables to connect the Geiger tubes to the high voltage supply. Because of the high voltage, we were worried that there may be a risk of arcing between lines on the breadboard; however, this was not observed. A lesson that I have learned before but that was reinforced here is that a significant portion of research time is spent troubleshooting.

This class has improved my ability to create presentations showcasing my work and helped me with articulating ideas that I understand into a form that someone else can understand, mainly in reference to the coincidence detector. It took me some time to get to the point where I understood the circuit enough to be able to explain it to others. It has also helped me to remember many things that I learned in Electronics that I had not used since. I have done research before, but this project seemed much closer to my interests than my previous work has. After this class, I am excited to continue working on this project and to get to grad school where more opportunities await.

References

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[3] Muon Students, http://scipp.ucsc.edu/outreach/TW/MuonStudents.ppt

[4] L. Liu, The Speed and Lifetime of Cosmic Ray Muons (2007), http://web.mit.edu/lululiu/Public/pixx/not-pixx/muons.pdf

[5] K. Raghav, Looking at Cosmic Muons to verify Einstein's Special Relativity (2010), http://www.cornellcollege.edu/physics-and-engineering/research/Student-Projects/Cosmic-Ray-Muons/Cosmic-Ray-Muons.html

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[9] Cosmic Ray (Muon) Detector using Geiger-Müller Tubes (2009), http://www.hardhack.org.au/geiger_muller_detector

[10] Cosmic Ray (Muon) 81 (9x9) Pixel Hodoscope (2011), http://www.hardhack.org.au/cosmicpixel

[11] T. Finley, Two's Complement (2000), http://www.cs.cornell.edu/~tomf/notes/cps104/twoscomp.html