Problem of the Block
Block 6

The game of Sprouts

The game of Sprouts was invented nearly 50 years ago. It is a ‘simple’ game played with pen and paper for two players. The game starts with a number of nodes drawn on the paper. A turn consists of connecting two nodes with an edge and placing a new node at the midpoint of the edge. Edges are not allowed to cross and each node can have at most 3 edges emanating from it. It is permissible to connect a node to itself with an edge (this counts as two edges emanating from the node). The player who makes the last permissible move wins.

Figure 1: (left) A starting configuration with 3 nodes. (right) The board after player 1 makes a move (note: you do not need to color or label nodes when playing)

Figure 2: The board after each player has made two moves. Nodes labeled A, B and 3 can no longer have any new edges.

Questions

1. Consider a game of Sprouts that starts with \( n \) points. For which \( n \) is the game guaranteed to end? Can you come up with the tightest upper and lower bounds on the number of moves it will take to complete the game for these \( n \)? Can you give an example where it does not terminate for the other \( n \) values?

2. Now consider a game of Sprouts that starts with 3 points but instead of allowing at most 3 lines to leave each node we allow 4. Is the game guaranteed to end? Prove it or provide a counterexample to show it does not.

3. In addition to the above two problems, pose a problem of your own related to sprouts. Can you solve it? Students who submit solutions (regardless of correctness) and a pose a question of their own will get access to the other student-posed questions and can attempt these problems for bonus points.

The Math Club will be hosting a GAME NIGHT February 13. Come play games - we will not make you think about the math behind them!

Turn in solutions to Dr. Skorczewski in Law 204 or by email at tskorczewski@cornellcollege.edu by March 16. Solutions for only one of the questions or partial solutions will receive credit (and are encouraged!). You can turn in solutions in pieces as you create them. For more information about the Problem of the Block see http://www.cornellcollege.edu/mathematics/problem-of-the-block/index.shtml.