Problem of the Block - Block 6

Seervada Park has recently been set aside for a limited amount of sightseeing and backpack hiking. Cars are not allowed into the park, but there is a narrow, winding road system for trams and for jeeps driven by the park rangers. This road system is shown in Figure 1, where location O is the entrance to the park, T is a fantastic scenic overlook, and the other letters denote ranger stations. The numbers denote the distance in miles between locations. A small number of trams are available to transport visitors from the entrance to the scenic overlook and back.

Problem 1

Determine which route from the park entrance O to the scenic overlook T has the shortest distance for the tram operations.

Problem 2

Telephone lines must be installed under the roads to establish communication among all the stations (including O and T). Because the installation is expensive and disrupts the natural environment, lines will be installed under just enough roads to provide some connection between every pair of stations. Where should the lines be laid to accomplish this with a minimum total number of miles of telephone line?

Problem 3

The third problem is that more people want to take the tram ride from the park entrance to the scenic overlook than can be accommodated during the peak season. To avoid unduly disturbing the ecology and wildlife of the region, a strict ration has been placed on the number of tram trips than can be made of each of the roads per day. (Since each tram will return on the same route as it took to the scenic overlook, we will focus on outgoing trips only.) The strict upper limits placed on the outgoing trips from each station is shown in Figure 2. For example, only 1 outgoing trip is allowed from A to B each day, one other outgoing trip is allowed from B to A, and 4 outgoing trips from C to E are allowed. (Remember we are only focused on trips to the scenic overlook; the return trips are implied.) Given the limits, one *feasible solution* is to send seven trams per day, with five using route $O \to B \to E \to T$, one using route $O \to B \to C \to E \to T$, and one using route $O \to B \to C \to E \to D \to T$. Note that this solution blocks the use of any routes that contain $O \to C$. Your task is to find the combination of routes that maximize the number of tram trips to the scenic overlook each day.

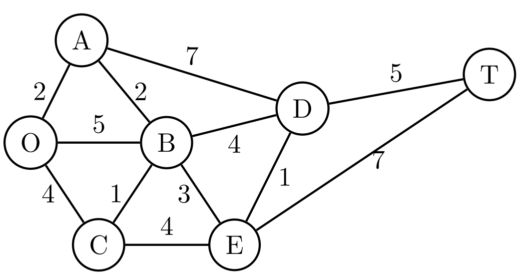

Figure 1: Use for problems 1 and 2

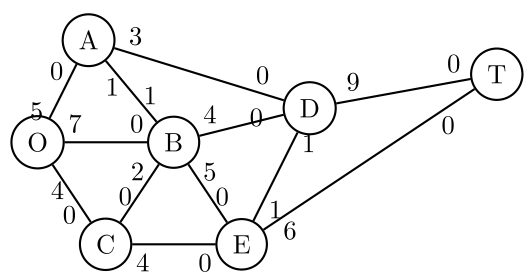

Figure 2: Use for problem 3

These types of problems come from Graph Theory and Operations Research. You may see more math like this next year in MAT 255 Graph Theory & Networks (block 7) and MAT 257 Operations Research (block 2) . For more information about these problems or if you are interested in pursuing a capstone similar to this see Dr. Freeman and Dr. S!

Turn in solutions to Dr. S in Law 204 or by email at tskorczewski@cornellcollege.edu by March 19.Partial solutions will receive credit (and are encouraged!). You can turn in a solution to just one question and turn in a solution to another question on a different day. The winning solution which earns the bonus points for the yearly competition will be the submission that is the best written, not necessarily the first. Submitting solutions to the Problem of the Block may earn culture points toward the math major. For more information about the Problem of the Block and to print off your own copy visit `http://www.cornellcollege.edu/mathematics/problem-of-the-block/index.shtml`.