A graph theory problem

A complete graph of order $n$ ($K_n$) is a set of $n$ vertices where each vertex is connected to every other vertex by an edge. The complete graphs of order 3 ($K_3$) and order 4 ($K_4$) are shown below.

Consider a complete graph of order $n$. Draw each edge of the graph with either a red pencil or a blue pencil. How large does $n$ need to be to guarantee that you will have drawn either a red triangle or a blue triangle? Note it is possible to draw a red triangle on $K_3$ by choosing all edges to be red, but it is not guaranteed since you could also choose one edge to be blue. We want to know how large $n$ needs to be to ensure a red or blue triangle regardless of the choices of we make when coloring the edges of the graph.

Now consider coloring the edges red, blue or green. How large does $n$ need to be ensure a red, blue or green triangle? In your solutions, make sure to include the logic you used to arrive at your values of $n$.

In addition to the above two problems, pose a problem of your own related to the ideas presented above. Can you solve it? Students who submit solutions (regardless of correctness) and a pose a question of their own will get access to the other student-posed questions and can attempt these problems for bonus points.

Turn in solutions to Dr. Skorczewski in Law 204 or by email at tskorczewski@cornellcollege.edu by 2/8/2015. Solutions for only one of the questions or partial solutions will receive credit (and are encouraged!). For more information about the Problem of the Block see http://www.cornellcollege.edu/mathematics/problem-of-the-block/index.shtml.