Problem of the Block Block 4



Overlapping circles

From the 1982 Math Olympiad in Bulgaria

In the plane, n circles of unit radius are drawn with different centers. Of course, overlapping circles partly cover^{*} each other's circumferences. A given circle could be so overlaid that any uncovered parts of its circumference are all quite small; that is, it might have no sizable uncovered arcs at all. However, this can't be true of every circle; prove that some circle must have a continuously uncovered arc which is at least 1/nth of its circumference.

(* Think of 'covered' as a reciprocal action. If circle A covers part of circle B, then circle B is also covering part of circle A.)

Turn in solutions to Dr. Skorczewski in Law 204 or by email at tskorczewski@cornellcollege.edu by January 15, 2017. Partial solutions will receive credit (and are encouraged!). You can turn in a solution to just one question and turn in a solution to another question on a different day. The Math Faculty can give hints or pose leading questions to help you get started or get past a hump. The winning solution which earns the bonus points for the yearly competition will be the submission that is the best written, not necessarily the first. Submitting solutions to the Problem of the Block may earn culture points toward the math major. For more information about the Problem of the Block and to print off your own copy visit http://www.cornellcollege.edu/mathematics/problem-of-the-block/index.shtml.