

# Simulation of a Queue

CSC131 Privacy, Piracy, and the Public Good

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In this exercise, I would like you to use the Excel spreadsheet software to simulate a waiting line (a "queue") in a bank. We shall assume that customers arrive in the bank one at a time and that there is just one teller to serve the customers.

This simulation is controlled by two parameters...

- The "Mean Interarrival Time" is the average time that elapses between the arrival of one customer and the arrival of the next customer.
- The "Mean Required Service Time" is the average time each customer requires of the teller.

You must choose values for these two parameters—any positive numbers will do. Clearly, if customers arrive much more quickly than the teller can serve them, a long waiting line will develop. If the teller is able to serve customers much more quickly than they arrive, each customer will get prompt attention. In our simulation, customers will arrive at random intervals and will require an amount of service from the teller that will also vary in a random way. Our goal is to measure the way in which waiting times vary as a function of the Mean Interarrival Time and the Mean Required Service Time.

I would like you to create a spreadsheet that computes several values for each simulated customer of the bank.

1. The "Interarrival Time of Customer # $n$ " is the amount of time that elapses between the arrival of the customer who precedes Customer # $n$  and the arrival of Customer # $n$ .

$$\text{InterarrivalTime}[n] = -\text{MeanInterarrivalTime} \times \ln(\text{rand}())$$

The Interarrival Time of Customer # $n$  will in every case be a positive number. There is no upper bound on its value but smaller values are more likely than larger ones.

The  $\text{rand}()$  function is one of the functions included in Excel's menu of functions. It produces a number in the range  $[0, 1]$  with an equal probability of returning any value in that range. Note that the logarithm function  $\ln()$  is undefined when  $x$  is zero or negative.

- The "Required Service Time of Customer #n" is the amount of time that Customer #n requires of the teller and is a measure of much business the customer has brought to the bank on this day. It also is a random positive number with no upper bound but with a greater probability of having a small value than a large value.

$$RequiredServiceTime[n] = -MeanRequiredServiceTime \times \ln(rand())$$

- The "Arrival Time of Customer #n" is just the sum of the interarrival times of Customer #n and all previous customers. It is the time customers and the teller see on the clock when the  $n^{th}$  customer enters the bank and joins the back end of the queue.

$$ArrivalTime[n] = ArrivalTime[n - 1] + InterarrivalTime[n]$$

- Customer #n gets the attention of the teller either when the teller finishes serving that customer who was immediately in front of Customer #n in the queue or when Customer #n arrives in the bank. The second case occurs only when Customer #n arrives in the bank after the teller has finished serving all customers who arrived before Customer #n.

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if (ArrivalTime[n] < TimeServiceCompleted[n - 1]) then
    TimeServiceBegins[n] = TimeServiceCompletes[n-1]
else
    TimeServiceBegins[n] = ArrivalTime[n]

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There is an "if" function in Excel's menu of functions.

- Computing the time at which the teller finishes serving Customer #n is straight-forward.

$$TimeServiceCompletes[n] = TimeServicesBegins[n] + RequiredServiceTime[n]$$

- The time each customer spends waiting in line is also easily computed.

$$WaitTime[n] = TimeServiceBegins[n] - ArrivalTime[n]$$

I would like you to simulate a queue with several different values of the MeanInterarrivalTime and MeanRequiredServiceTime and in each case compute the minimum, maximum, and mean wait times.